National Science and Mathematics Olympiad

Second Stage- Physics

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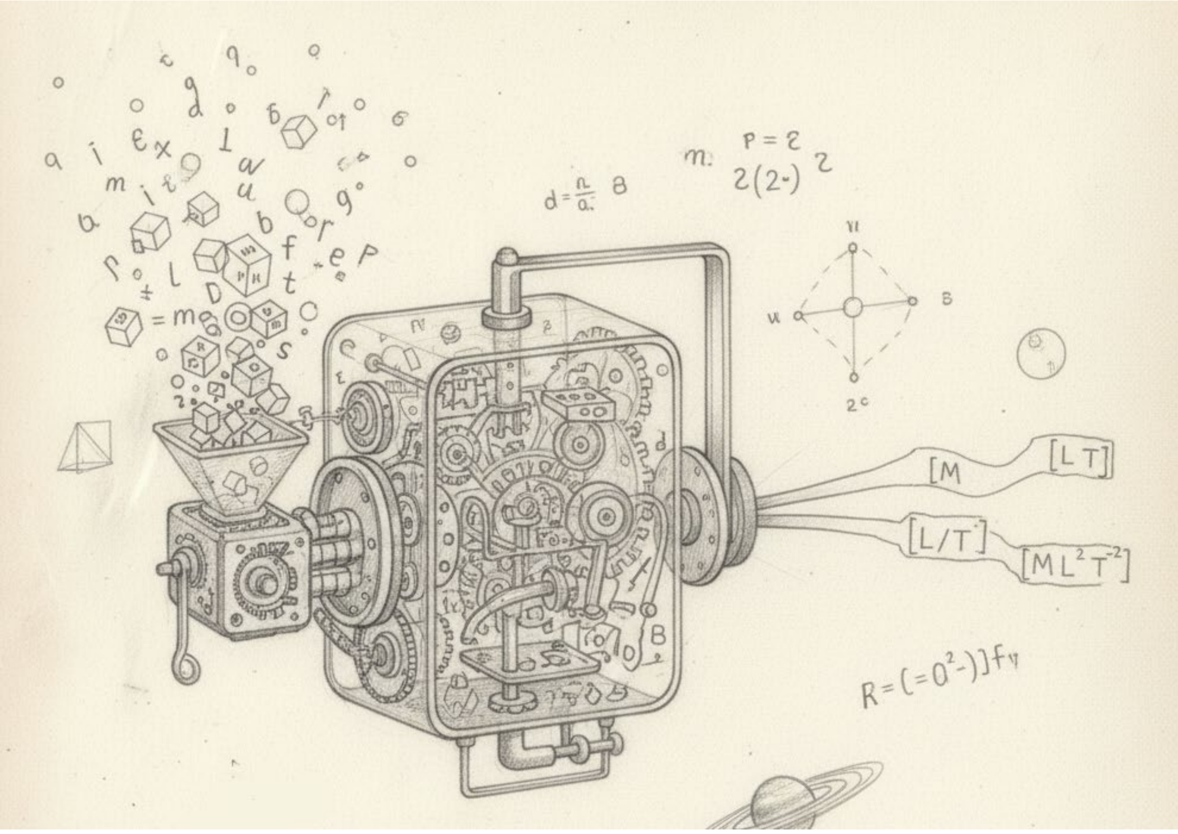
1 Chapter 1: DIMENSIONAL Analysis in Physics



Dimensional analysis is a mathematical–physical method used to verify the correctness of physical equations and to derive relations among different quantities. It is based on the idea that any physical quantity can be expressed in terms of a set of fundamental dimensions that constitute it.

For example, distance, displacement, height, and the circumference of a circle all share the same dimension-length, denoted by the symbol [L]. Units of length may include m, cm, km, inch, or feet; despite their differences, they all represent the same physical dimension. Similarly, the dimension of velocity is $[L\ T^{-1}]$, regardless of the unit used for measurement.

Note: The letter L stands for length, and placing it in brackets, [L], indicates that we refer to the dimension of length, not its numerical value.

1.1 Dimensions of Some Physical Quantities

Fundamental Quantities	Dimension	Derived Quantities	Dimension
Length	[L]	Area	$[L^2]$
Mass	[M]	Volume	$[L^3]$
Time	[T]	Velocity	$[LT^{-1}]$
Electric Current	[I]	Acceleration	$[LT^{-2}]$
Temperature	[Θ]	Force	$[MLT^{-2}]$
Amount of Substance	[N]	Density	$[ML^{-3}]$
Luminous Intensity	[J]	Frequency	$[T^{-1}]$

1.2 Fundamental Rules

- Quantities can be added or subtracted only if they have the same dimensions.
- When quantities are multiplied or divided, the exponents of their dimensions are added or subtracted respectively.

For example, we can find the force dimension this way:

Force = Mass × Acceleration

$$[F] = [M][LT^{-2}] = [MLT^{-2}]$$

Note: Numerical constants such as ½ or π are dimensionless.

Example 1-1

Finding the Dimensions of Areas

1. Area of a Circle

$$A = \pi r^2$$

Radius *r* has dimension [L], and π is dimensionless.

$$[A] = [L^2]$$

Hence, the dimension of the area of a circle is $[L^2]$.

2. Area of a Triangle

$$A = \frac{1}{2}bh$$

Both base *b* and height *h* have the dimension [L], while ½ is dimensionless.

$$[A] = [L][L] = [L^2]$$

3. Area of a Rectangle

$$A = l \times w \Rightarrow [A] = [L^2]$$

General Result: All types of area share the same physical dimension, $[L^2]$, regardless of how the area is calculated.

1.3 Importance of Dimensional Analysis

- Checking the Validity of Physical Equations
  An equation is dimensionally correct if both sides have identical dimensions.

Example 1-2

Check the validity of the equation

$$d = v_i t + \frac{1}{2} a t^2$$

The left-hand side dimension is $[\mathrm{d}]$ is $[\mathrm{L}]$

The right-hand side:

$$[\mathrm{v_i}][\mathrm{t}] = [\mathrm{LT^{-1}}][\mathrm{T}] = [\mathrm{L}]$$
$$[a][\mathrm{t^2}] = [\mathrm{LT^{-2}}][\mathrm{T^2}] = [\mathrm{L}]$$

Therefore, the equation is dimensionally consistent, but if the dimensions differ between the two sides of the equation, then the equation is wrong.

- Deriving Relations Between Physical Quantities
  Dimensional analysis helps estimate the form of relationships between variables, even when the exact equation is unknown.

If the dimensions on both sides of the equation are not the same, then the equation is incorrect.

Example 1-3

Estimating the Time of Free Fall

Suppose the laws of free fall are unknown. How can we determine the factors affecting the fall time t using dimensional analysis?

Step 1: Assume the General Relationship

$$t \propto m^{\alpha} h^{\beta} g^{\gamma}$$

where

m = mass, h = height, g = gravitational acceleration.

A common misconception is that it depends on m, so we keep m in the expression for testing.

Step 2: Substitute Dimensions

Quantity	Symbol	Dimension
Time	t	[T]
Mass	m	[M]
Height	h	[L]
Gravity	g	$[L\,T^{-2}]$

Substitute:

$$[T] = [M]^{\alpha}[L]^{\beta}[LT^{-2}]^{\gamma}$$
$$[T] = [M]^{\alpha}[L]^{(\beta+\gamma)}[T]^{-2\gamma}$$

Step 3: Equate Exponents of Each Dimension

For Mass [M]:

$$0 = \alpha \Rightarrow \alpha = 0$$

→ Time of fall does not depend on mass.

For Time [T]:

$$1 = -2\gamma \Rightarrow \gamma = -\frac{1}{2}$$

For Length [L]:

$$0 = \beta + \gamma \Rightarrow \beta = \frac{1}{2}$$

Step 4: Final Relation

$$[T] = [M]^{0}[L]^{1/2}[T]^{-1/2} \Rightarrow t \propto \sqrt{\frac{h}{g}}$$

Thus,

- The time of fall depends on the height and the acceleration due to gravity,
- and does not depend on mass (neglecting air resistance).

If we replace the proportionality with equality using a constant k:

$$t = k\sqrt{\frac{h}{g}}$$

From the kinematic equation $t = \sqrt{\frac{2h}{g}}$,

we find that $k = \sqrt{2}$, which is a dimensionless constant.

Example 1-4

Verifying the Kinetic Energy Equation

$$E_k = \frac{1}{2}mv^2$$

Dimensions:

$$[E_k] = [M][LT^{-1}]^2 = [ML^2T^{-2}]$$

This matches the dimension of energy (joule), confirming that the equation is dimensionally valid.

1.4 Additional Problems

1. Using dimensional analysis, find the dimension of pressure from: $P = \frac{F}{A}$

2. By using dimensional analysis, show the quantities with which the period of a simple pendulum is related.

   The period depends on the length $[L]$and the gravitational acceleration $[LT^{-2}]$.

3. Any object can escape a planet's gravitational field when launched at a certain speed called escape velocity. The escape velocity v_{esc} from a planet depends on the planet mass M, its radius R, and the universal gravitational constant G.

   Using dimensional analysis to determine how these quantities combine.

   Hint: Use Newton's law of universal gravitation $F = \frac{GMm}{R^2}$ to find the dimensions of G.

   A) $v \propto \frac{GM}{R^2}$
   B) $v \propto \sqrt{\frac{GM}{R}}$
   C) $v \propto \frac{G^2M^2}{R}$
   D) $v \propto \sqrt{\frac{GR}{M}}$

4. When you press the spring, it stores energy. The elastic potential energy Ustored in a spring depends on the spring constant $k$and the extension x.

   Using dimensional analysis to determine which relationship is correct.

Hint: From Hooke's Law, the force on a spring is $F = kx$, use this relation to find the dimensions of $k$before solving.

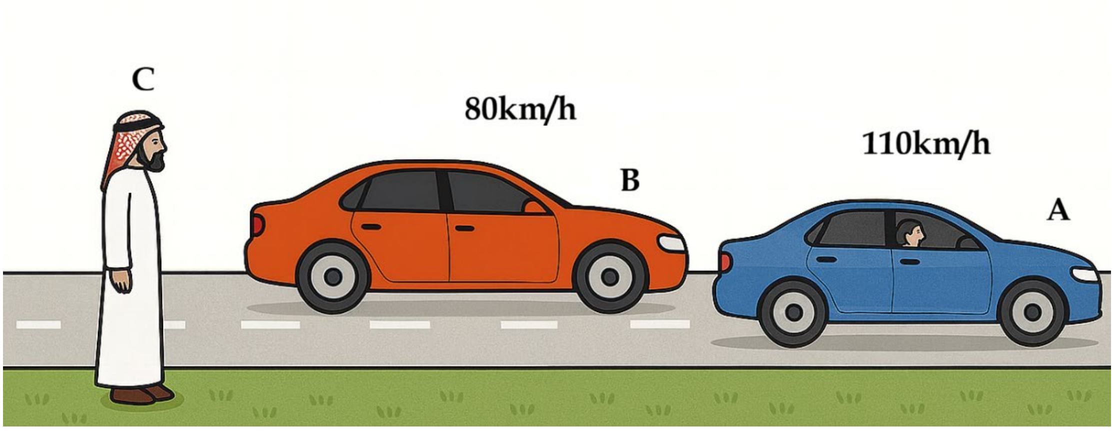
A) $U \propto kx$
B) $U \propto kx^2$
C) $U \propto k^2x$
D) $U \propto k^2x^2$

2 Relative Motion

2.1 relative velocity in one dimension

how the measurements made by two observers, one moving relative to the other—are related. We will see that the displacement, velocity, and acceleration measured for the same object may differ depending on the reference frame in which the observations are made. We deal with relative motion daily in our lives. A passenger in a car sees the streetlights beside the road moving backward, and the book that appears stationary to you is moving at high speed due to the Earth's motion.

We simply mean by a reference frame: An observer of an event, a value of a measured velocity of a motion of an object depends on reference frame which it is measured from, or in other words (the observer).



For example, when two cars are moving on a straight road and in one direction, and: the velocity of the first car A: 110 km/h, the velocity of the second car: 80 km/h. Then two observers can be distinguished: An observer $\boldsymbol{C}$ standing on the sidewalk: sees that the velocity of car A is: 110 km/h. We call it: the relative velocity of the car $\boldsymbol{A}$relative to the ground: $\boldsymbol{v}_{AC} = 110$ km/h. While an observer is riding in the car $\boldsymbol{B}$, he sees that the velocity of the car $\boldsymbol{A}$is 30 km/h. We call it: the relative velocity of the car $\boldsymbol{A}$relative to the car$\boldsymbol{B}$: $\boldsymbol{v}_{AB} = 30$ km/h.

Therefore:

Relative velocity: is the measured velocity of an object relative to a given reference frame.

Relative motion: It is the motion that is described in respect to a given reference frame.

Observing the previous figure: we find that the relationship between the displacements is as follows:

$$\vec{x}_{AC} = \vec{x}_{AB} + \vec{x}_{BC}$$

differentiation of the function $:\frac{d}{dt}x_{AC} = \frac{d}{dt}x_{AB} + \frac{d}{dt}x_{BC}$

$$\vec{v}_{AC} = \vec{v}_{AB} + \vec{v}_{BC} \qquad \vec{v}_{AB} = \vec{v}_{AC} - \vec{v}_{BC}$$

$$\vec{v}_{AB} = \vec{v}_{AC} + \vec{v}_{CB}$$

Relative velocity can be calculated as the sum of two velocities (note how the symbols are broken down in the equation): $\vec{v}_{AB} = \vec{v}_{AC} + \vec{v}_{CB}$ The law of relative velocity:

Applying to the previous example: The car's velocity $\boldsymbol{A}$relative to the car $\boldsymbol{B}$is the resultant of: The velocity of the car $\boldsymbol{A}$relative to the observer $\boldsymbol{C}$and the velocity of the observer $\boldsymbol{C}$ relative to the car $\boldsymbol{B}$: $\vec{v}_{AB} =$ 110 km/h − 80 km/h=30 km/h

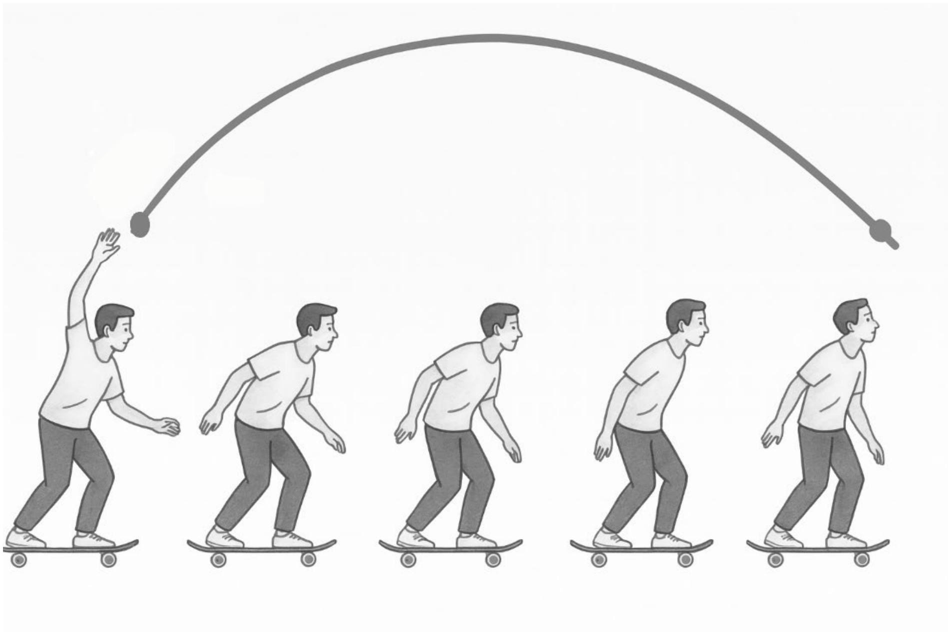
Note that the observer's velocity $\boldsymbol{C}$relative to the car$\boldsymbol{B}$ is negative because the observer $\boldsymbol{C}$appears to be moving to the left (negative direction) relative to the car$\boldsymbol{B}$.

Assuming that car A is accelerating and the speed of car B is constant, and by differentiating the velocity equation with context to time

$$\frac{d}{dt}\vec{v}_{AB} = \frac{d}{dt}\vec{v}_{AC} + \frac{d}{dt}\vec{v}_{CB} = \vec{a}_{AB} = \vec{a}_{AC} \text{ because } \frac{d}{dt}\vec{v}_{CB} = 0$$

Important Note:

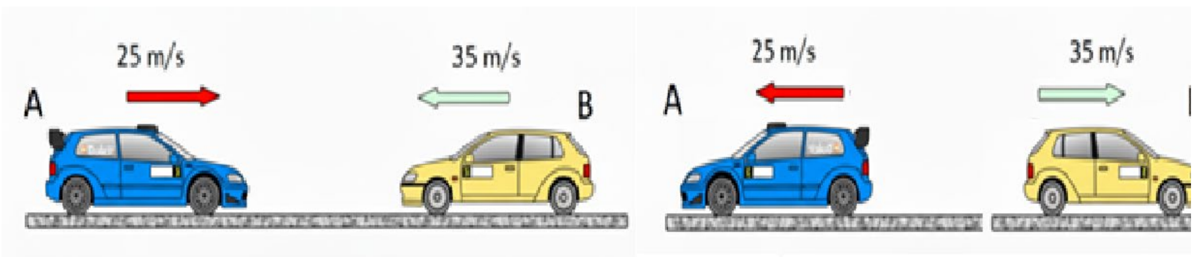
Observers on different reference frames that move at constant velocity relative to each other will measure the same acceleration for a moving particle.



The observer on the moving skateboard throws a ball upwards and sees it rise and fall in a straight line, while the stationary observer sees a parabolic trajectory for the same ball.

Example 2-1

Two cars are moving as in figure, Find the speed of the car $\boldsymbol{A}$ relative to the car $\boldsymbol{B}$.



solution

Taking rightward as positive direction:

When the two cars move towards each other

$$v_{a/g} = +25\ m/s, v_{b/g} = -35\ m/s$$

$$v_{a/b} = v_{a/g} + v_{g/b}$$

$$v_{a/b} = 25 + [-(-35)] = 60\ m/s, rightward$$

When the two cars move away from each other

$$v_{a/g} = -25\ m/s, v_{b/g} = +35\ m/s$$
$$\boldsymbol{v}_{a/b} = \boldsymbol{v}_{a/g} + \boldsymbol{v}_{g/b}$$
$$v_{a/b} = -25\ m/s, v_B = +35\ m/s$$
$$v_{a/b} = -25 - 35 = -60\ m/s\ (or\ 60\ m/s\ to\ the\ left)$$

Notice: $\boldsymbol{v}_{g/b} = -\boldsymbol{v}_{b/g}$

Example 2-2

A police car traveling at 95.0 km/h is traveling west, chasing a motorist traveling at 80.0 km/h. If they were originally 250 m apart, at what time interval would the police car overtake the motorist?

Solution

Given:

$$Police\ car\ speed,\ v_{p/g} = 95.0\ km/h$$
$$Motorist\ speed, v_{m/g} = 80.0\ km/h$$
$$Initial\ separation, d = 250\ m$$

Relative speed

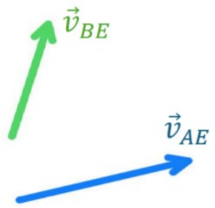
$$\boldsymbol{v}_{p/m} = \boldsymbol{v}_{p/g} + \boldsymbol{v}_{g/m} = 95km/h - 80km/h = 15km/h = 4.17\ m/s$$

Time to overtake

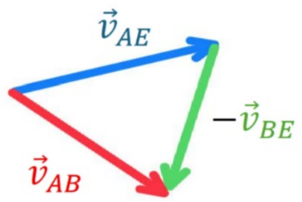
$$t = d\ /\ v_{\frac{p}{m}} = 250\ m/\ 4.17\ s = 59.95\ s = 6.0 \times 10^1 s$$

2.2 Relative Velocity In 2 Dimensions

Two cars are moving at the speeds shown in the figure, with speeds drawn relative to an observer on the ground.



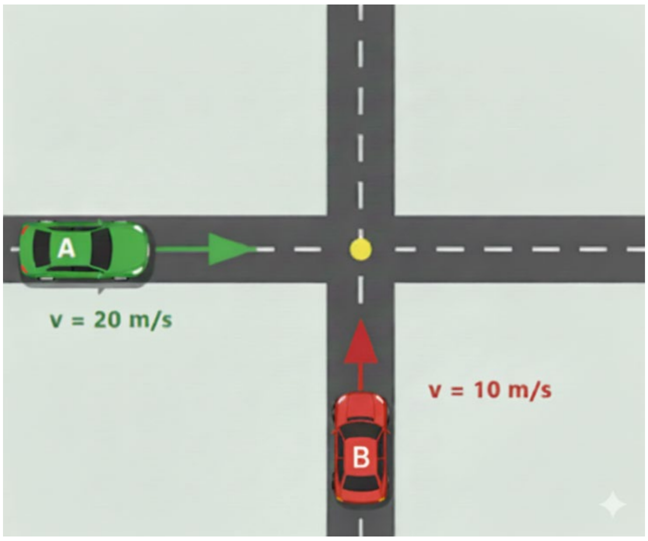
If we want to calculate the velocity of car A relative to car B, that is, as seen by an observer located in B, we use the main equations:



$$\vec{v}_{AB} = \vec{v}_{AE} + \vec{v}_{EB}$$
$$\vec{v}_{AB} = \vec{v}_{AE} - \vec{v}_{BE}$$

Example 2-3

Two cars are moving as in figure, Find the velocity of car A relative to car B



solution

$$\boldsymbol{v}_{a/b} = \boldsymbol{v}_{a/g} + \boldsymbol{v}_{b/g}$$

We observe that the two velocities are perpendicular, so we use Pythagoras' theorem to add the vectors in two dimensions.

$$v_{A/B} = \sqrt{(20)^2 + (10)^2} = 22.4\ m/s$$

$$\theta = \tan^{-1}(\frac{20}{10}) = \tan^{-1}(2) = 63.4°$$

Direction: 63.4° west of north

Example 2-4

A girl on a small raft across a river. She paddles is moving to the riverbank with a speed of $0.900\ m/s$. The river current carries the raft downstream at a speed of $1.40\ m/s$.

(a) What is the girl's resultant speed relative to the riverbank?

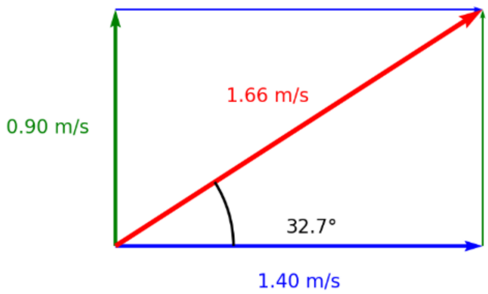
(b) At what angle relative to the river current does she move?

Solution

(a) Resultant Speed:

$$\vec{v}_{gb} = \vec{v}_{gr} + \vec{v}_{rb}$$

$$v_{gb} = \sqrt{(0.90)^2 + (1.40)^2} = \sqrt{0.81 + 1.96} = \sqrt{2.77}$$
$$\approx 1.66 \text{ m/s}$$



(b) Angle:

$$\tan\ \theta = \frac{0.90}{1.40}$$
$$\theta = \tan^{-1}(0.643) \approx 32.7°$$

2.3 Additional problems

1: A boat heads due north across a river with a speed of $12.0\ km/h$ relative to the water. The water in the river flows due east at $4.00\ km/h$ relative to the Earth.

a) Determine the velocity of the boat relative to an observer on the riverbank.

b) If the river is $2.50\ km$ wide, find the time required for the boat to cross.

2: If the boat of the preceding Exercise travels with the same speed of $12.0\ km/h$ relative to the river and is to travel due north, as shown in Figure.

a) What should its heading be?

b) If the river is $2.50\ km$ wide, find the time that the boat needs to cross it.

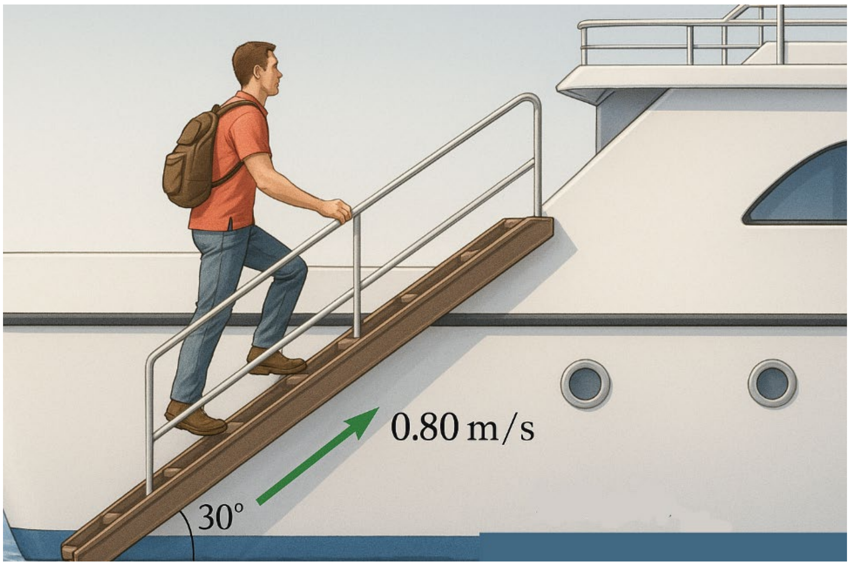
3: A plane is flying due east, but the pilot must steer the plane slightly south of east to counteract a steady wind blowing toward the northeast. The plane has a velocity relative to the wind of $240\ km/h$, directed at an angle θ south of east. The wind has a velocity relative to the ground of $70.0\ km/h$, directed $25.0°$ east of north.

(a) What is the magnitude of the velocity of the plane relative to the ground?

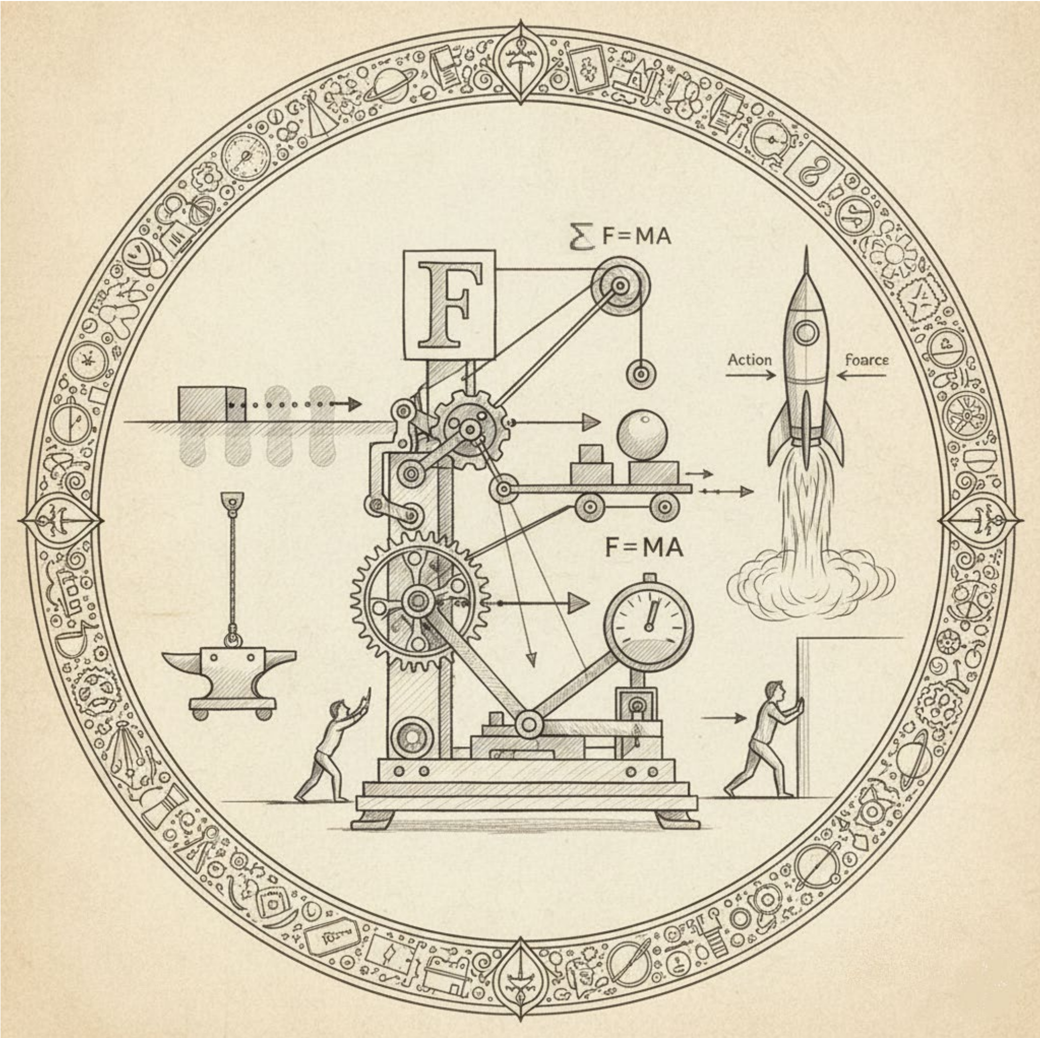
(b) What is the angle θ the pilot must point the plane south of east to ensure it travels due east?

4: An airplane is flying due east at a speed of 750 km/h. What must be the speed of a second airplane heading $60°$east of north if it always appears due north from the first airplane?

A person on a ferry boat walks up a ramp at a speed of $0.80\ m/s$ relative to the boat. The ramp makes an angle of $30°$ above the deck, and the ferry itself is moving to the east across calm water with a speed of $2.20\ m/s$. What is the velocity of the person relative to the water?



3 Dynamics: Newton's Laws



In previous chapters on mechanics, we described motion in terms of displacement, velocity, and acceleration without investigating the underlying causes. We did not ask ourselves: What causes motion? Why does one object remain stationary while another moves? What makes an object accelerate or decelerate?

To answer these questions, we must study force and relate it to motion - a field known as dynamics. In this chapter, we will define force and discuss its various types, including weight, normal forces, and friction forces. We will also examine Newton's three laws of motion, formulated over three centuries ago by Isaac Newton, which marked a turning point in our understanding of motion. Finally, we will apply these laws to real-world problems to solidify our comprehension.

3.1 The Concept of Force

The force (F) is one of the fundamental concepts in physics. It describes an interaction that can change the state of motion or the shape of an object. When a force acts on a stationary object, it can cause it to move, and when it acts on a moving object, it can change its speed or direction. A force is not a tangible thing, but an external influence whose effect appears through the acceleration or deformation of the object.

Force is measured in the International System of Units (SI) using the newton (N), and it is a vector quantity, meaning it has magnitude, direction, and a point of application. Forces in nature vary, including gravity that pulls objects toward the Earth, tension in ropes, the normal force from surfaces, friction that resists motion, and the applied force resulting from direct pushing or pulling.

The study of forces forms the foundation of dynamics, as it is closely related to Newton's laws of motion, which explain how and why objects move or remain at rest.

- The force is a vector quantity, that mean it has a direction and a value, and it follows the rules of vector addition discussed before, we can represent any force on a diagram by an arrow, the direction of the arrow is the direction of the force, and its length is drawn proportional to the magnitude of the force.

- Forces do not always cause motion, however. For example, as you sit reading this book, a gravitational force acts on your body and yet you remain stationary. As a second example, you can push (in other words, exert a force) on a large boulder and not be able to move it.
- Force is measured in newton (N) in the International System of Units SI, and in (dyne) in the French system CGS: 1 dyne=10^{-5} N

Classifications of Forces

- Contact forces: Require contact between two bodies, such as tension forces.
- Field forces: The effect occurs through a field without direct contact, such as electrical attraction forces or gravitational attraction between masses.

Important Concepts:

System: It is an object or objects under study.

Environment or surroundings: everything that surrounds the system and Effects on it with forces.

Free-Body Diagram

It is prepared with the following steps A diagrammatic representation,

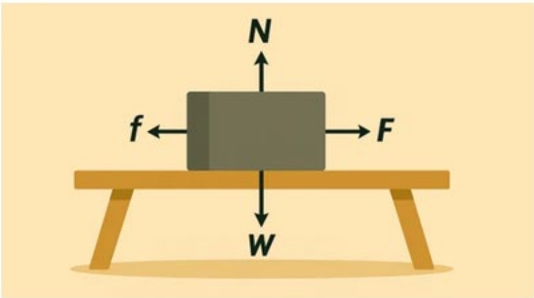
Abbreviate the system as a point and put it in the origin of the system coordinates.

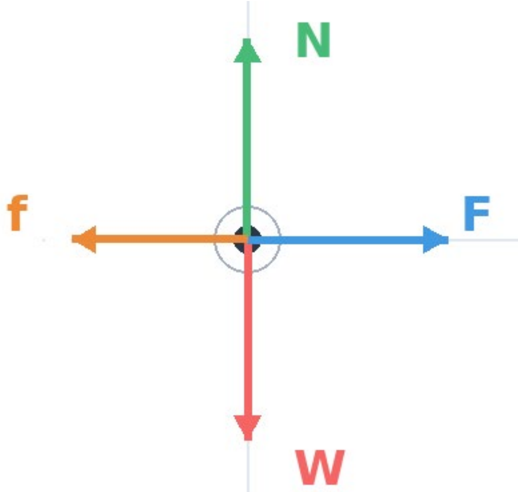
Draw the forces acting on the system as vectors, with care of lengths (proportional to values) and directions.

Forces in directions $+y, +x$: positive Force in directions $-y, -x$: negative

Example 3-1

The following figure shows a free-body diagram of the forces acting on a box moving to the right under the influence of a force pulling it to the right.





Note that we are only drawing the forces acting on the box, and we are not concerned with the forces acting on the table by the ground or by the box.

3.2 Newton's First Law

What is the relationship between force and motion?

Aristotle (384–322 BC) believed that a force is necessary to keep an object moving on a horizontal surface. He considered rest to be the natural state of an object and assumed that a body requires a continuous force acting on it to remain in motion. He also argued that the greater the applied force, the greater the object's speed.

Nearly two thousand years later, Galileo Galilei disagreed with Aristotle's view. He argued that the natural state of a body is to remain in uniform motion - that is, to move with a constant velocity - or to remain at rest if no external forces act upon it.

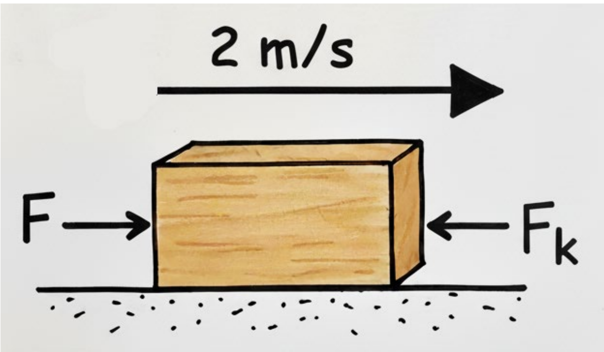
To illustrate, imagine an object placed on a perfectly smooth surface - a table coated with an ideal frictionless oil layer. Once the object is set in motion, it will continue moving at a constant speed without being affected by any frictional force. This brilliant idea of Galileo, the concept of eliminating friction, provided a profound foundation for understanding the physical world.

Building upon Galileo's insight, Sir Isaac Newton developed his monumental theory of motion, summarized in his three famous laws of motion presented in his great work "Philosophiea Naturalis Principia Mathematica" published in 1687.

Newton acknowledged Galileo's influence, as Newton's First Law of Motion closely aligns with Galileo's conclusions, and it is also known as the Law of Inertia.

An object at rest remains at rest, and an object in motion remains in motion at constant speed and in a straight line unless acted on by an unbalanced force.

Notice how Newton's first law clearly applies to the box when it moves at constant velocity in a straight line, and to the stationary vase.



And in both cases $\sum \vec{F} = 0$ and $a = 0$

$v = 0$ or $v = constamt$

The objects then are in: Transitional Kinetic Equilibrium.

3.2.1 Inertial Frames

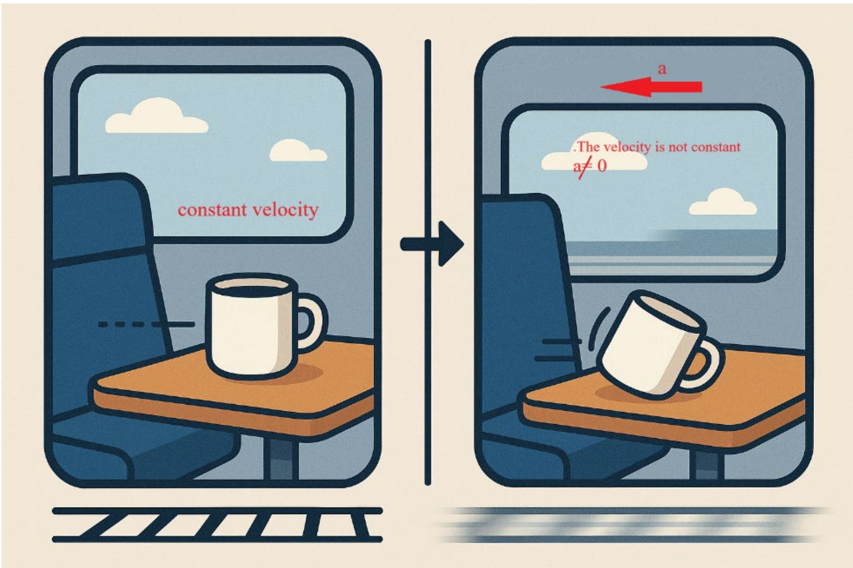
We simply mean by reference frame: the observer of an event, a special group of reference frames called inertial reference frames, which are non-accelerate.

Since Newton's first law relates only to objects without acceleration, it is only true in static frames.

Newton's first law is not held in every reference frame. For example, if your reference frame is an accelerating car, an object such as a cup resting on the dashboard may begin to move toward you (it stayed at rest as long as the car's velocity remained constant). The cup accelerated toward you, but neither you nor anything else exerted a force on it in that direction.

In accelerating reference frames, Newton's first law does not hold. Physics is easier in reference frames in which Newton's first law does hold, and they are called inertial reference frames (the law of inertia is valid in them). For most purposes, we usually make the approximation that a reference frame fixed on the Earth is an inertial frame. (This is not precisely true, due to the Earth's rotation, but usually it is close enough).

Any reference frame that moves with constant velocity (say, a car or a train) relative to an inertial frame is also an inertial reference frame. Reference frames where the law of inertia does not hold, such as the accelerating reference frames discussed above, are called non-inertial reference frames.



3.2.2 Inertia

The tendency of an object to remain in its state of rest or uniform motion in a straight line is called inertia.

For example, it is difficult to set a stationary object in motion, to stop a moving object, or to change its velocity sideways out of a straight-line path.

Inertia depends on the mass of the object; it increases as the mass increases.

Mass is an inherent property of an object and does not depend on the surrounding medium or on the method used to measure it.

Newton used the term mass as a synonym for “quantity of matter.” However, this notion is not very precise, since the concept “quantity of matter” itself is not well defined.

The following phenomena can be explained based on the concept of inertia:

When a car suddenly stops, passengers tend to lurch forward.

This happens because their bodies try to continue moving with the same speed and direction as before the car stopped — that is, their bodies resist the change in their state of motion.

When a tablecloth is pulled quickly from under dishes, the dishes remain almost in place.

This occurs because the dishes have inertia, which makes them resist the sudden change from rest.

When a stationary ball is kicked, a force is required to set it in motion.

The greater the mass of the ball, the harder it is to move, because its inertia is greater.

When moving a heavy object such as a water tank or a large box, it is difficult to start or stop its motion quickly. This is because inertia increases directly with mass.

3.3 Newton's second Law

Newton's first law explains what happens to an object when no forces act on it. It either remains at rest or moves in a straight line with constant speed. Newton's second law answers the question of what happens to an object that has a nonzero resultant force acting on it.

Newton's second law states that:

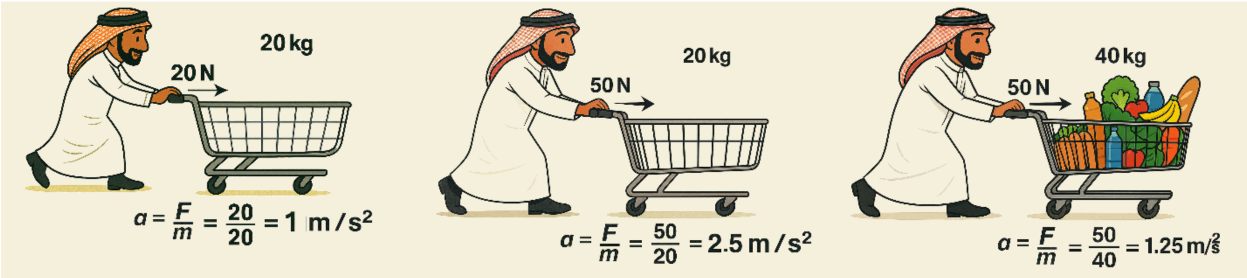
When viewed from an inertial reference frame, the acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass.

The equation:

$$\sum \boldsymbol{F} = m\boldsymbol{a}$$

$$\sum F_y = ma_y \ \sum F_x = ma_x$$

Note the proportionality in the example of a cart moving under a horizontal force on a surface of negligible friction, study the figure.



Important Notes

- We apply Newton's second law to the net force, not to the single forces.
- We assume that objects can be modeled as particles (point masses) so that we need not worry about rotational motion.

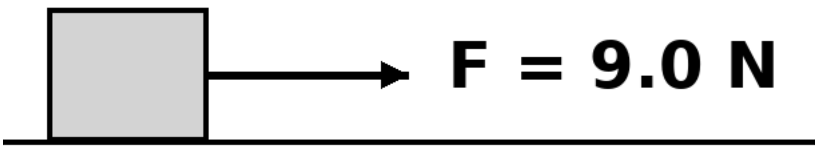
Think:

- Is the acceleration always in the direction of the net force? And why?
- Write newton (N) in terms of base units?
- A car is moving north at a constant speed, what is the direction of the net force acting on it?

Example 3-2

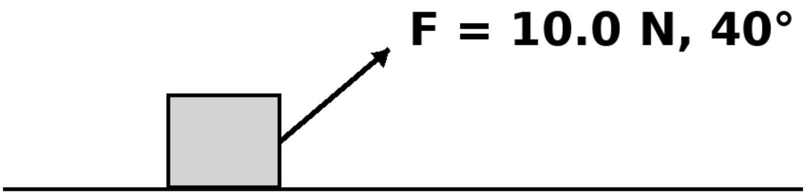
A box of mass $3.0\ kg$ moves horizontally on a smooth surface under the forces shown. Calculate the horizontal acceleration. (Neglect friction; consider only horizontal components.)

(a) Single horizontal force: $F = 9.0\ N$ to the right.



$a = F/m = 9.0 / 3.0 = 3.0\ m/s^2$ (to the right).

(b) Single force: $F = 10.0\ N\ at\ 40.0°$ above the horizontal.



$a = (F\cos 40°)/m = (10.0\cos 40°)/3.0 = 2.55\ m/s^2$ (to the right).

Example 3-3

What average net force is required to bring a 1500kg car to rest from a speed of 100km/h within 55 m?

Given:

$m = 1500\text{ kg}, v_i = 100\text{ km/h} = \frac{100\times1000}{3600} = 27.78\text{ m/s}, v_f = 0\text{ m/s}, d = 55\text{ m}$

Find acceleration using kinematics

$$v_f^2 = v_i^2 + 2ad$$
$$0 = (27.78)^2 + 2a(55)$$
$$a = -\frac{(27.78)^2}{2(55)} = -7.0\text{ m/s}^2$$

Find the net force

$$F_{\text{net}} = ma = 1500 \times (-7.02) = -1.0 \times 10^4\text{ N}$$

Exercise 1: A hockey puck of mass 0.40 kg slides on a smooth horizontal surface of ice. Two forces act on it: the first $F_1 = 7.5$ Nat a standard angle of $-30.0°$, and the second $F_2 = 6.0$ Nat a standard angle of $30.0°$. Find the acceleration of the puck using unit vectors.

3.3.1 Weight

It is the magnitude of the Earth's gravitational force on an object. Its direction is always toward the center of the Earth, meaning perpendicular to the Earth's surface.

$\boldsymbol{F_g = W = mg}$

Where:

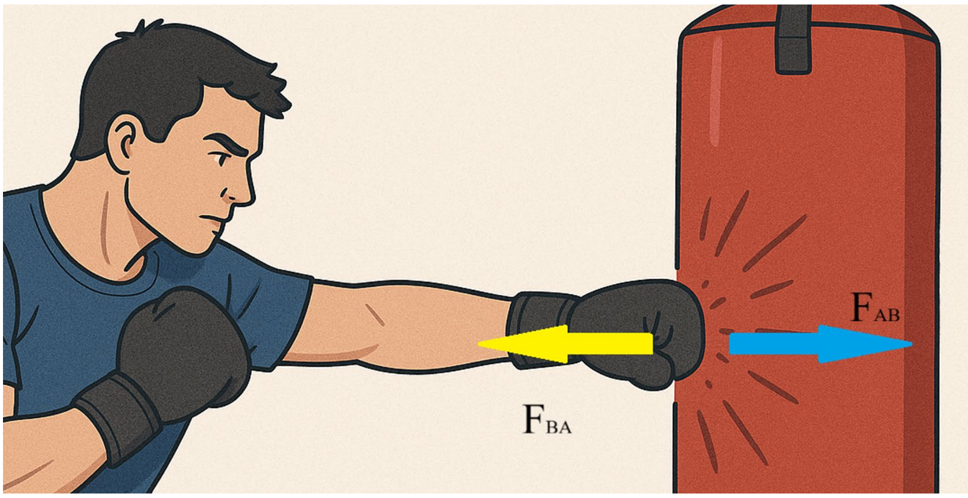
W: Weight   m: Mass of the object   g: Acceleration due to Earth's gravity

Remember that: The value of gravitational acceleration at the Earth's surface is approximately 9.8 m/s², and its magnitude decreases with altitude, meaning that weight decreases with altitude.

Important Note

Newton's second law, like the first law, is valid only in inertial reference frames, in the non-inertial reference frame of a car that begins accelerating, a cup on the dashboard starts sliding-it accelerates- even though the net force on it is zero. Thus $\sum \boldsymbol{F} = \boldsymbol{ma}$ does not work in such an accelerating reference frame.

3.4 Newton's Third Law



Newton realized that things are not so one-sided. True, the hammer exerts a force on the nail, but the nail evidently exerts a force back on the hammer as well, for the hammer's speed is rapidly reduced to zero upon contact. Only a strong force could cause such a rapid deceleration of the hammer. Thus, said Newton, the two objects must be treated on an equal basis.

Newton's Third law states that:

If two objects interact, the force F_{AB} exerted by object A on object B is equal in magnitude and opposite in direction to the force F_{BA} exerted by object B on object A

"To every action there is an equal and oppo site reaction

The equation

$$F_{AB} = -F_{BA}$$

The two forces are called action and reaction, and either of the two forces can be action or reaction.

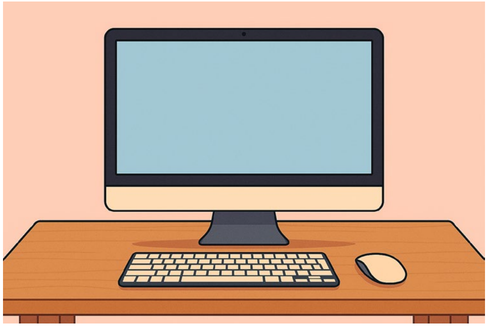
The two forces are equal in magnitude and opposite in direction.

Think: What is the sum of action and reaction forces, and why?

Results of Newton's third law:

forces always occur in pairs, or that a single isolated force cannot exist.

Check concept: When a computer monitor is at rest on a table, determine the forces acting on the monitor and the reaction to each one.



Think: if A large truck collides with a small racing car, which of them will be affected by more force

and why?

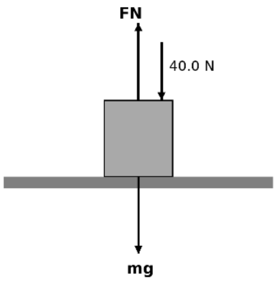
3.4.1 Normal Force

Definition: The force that the surface exerts on the object is placed on it

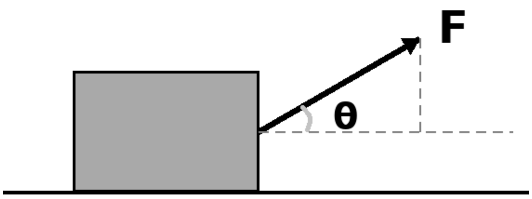
Direction: Perpendicular to the surface at the point of contact

Check concept: Find formula for calculating the normal force from weight and other forces in the following cases presuming that all surfaces are frictionless:

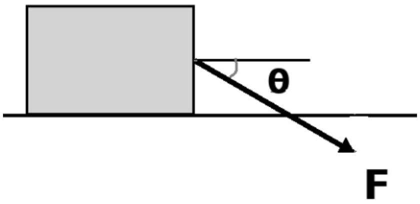
A box that is pushed down



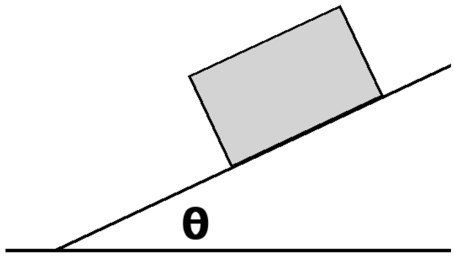
Object moving horizontally



Object moving horizontally



Object sliding on a frictionless inclined plane.



3.4.2 Elevator and Apparent weight

The person standing in the elevator is subjected to the influence of two forces: his weight mg and the reaction of the analog scale F_N . The analog scale measures the reaction F_N

Example 3-4

Find formula for calculating the apparent weight from the real weight in the following states:

a) Elevator is static

b) Elevator is moving at a constant speed.

c) Elevator is accelerating up

d) Elevator is accelerating down

$Using: g = 9.8\, m/s^2$

a) Elevator static ($a = 0$)

$$N = W = mg$$

b) Elevator moving at constant speed ($a = 0$)

$$N = W = mg$$

c) Elevator accelerating up ($a > 0$)

$$W = N = m(g + a)$$

d) Elevator accelerating down ($a > 0$ downward)

$$W = N = m(g - a)$$

Note: If $\boldsymbol{a} = \boldsymbol{g}$ downward (free fall), then $N = 0$ (weightlessness).

Important Notes

- Solving problems of incline is usually easier if we choose the x-y coordinate system so the x axis points along the incline (the direction of motion) and the y axis is perpendicular to the incline.

-Force of gravity is not perpendicular to the slope, gravity acts vertically downward toward the center of the Earth

Guidelines for Solving Problems Using Newton's Second Law:

Follow this method when dealing with problems involving Newton's laws:

- Draw a simple and accurate diagram of the problem.
- Isolate the object whose motion you are analyzing. Draw a free-body diagram for this object. For systems containing more than one object, draw a separate free-body diagram for each object. Do not include in the diagram the forces exerted by the object on its surroundings. Establish appropriate coordinate axes for each object, then find the components of the forces along these axes.
- Apply Newton's Second Law in component form.
- Solve the component equations for the required unknowns and remember that you must have the same number of equations as the number of unknowns.
- Verify that your results are consistent with the diagram.

3.5 Applications for Newton's Laws:

3.5.1 Acceleration of Multiple-Body Systems:

Example 3-5

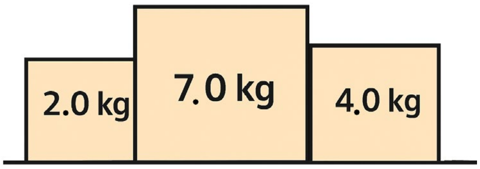
Three blocks are in contact on a smooth horizontal surface. Their masses are: $m_a = 2.0\ kg, m_\beta = 7.0\ kg, and\ m_c = 4.0\ kg$. A horizontal force of $20\ N$ acts on block $2.0kg$ to the right.

a) Find the acceleration of the system.

b) Determine the net force on each block.

c) Calculate the contact force between blocks A and B.

d) Calculate the contact force between blocks B and C.



Solve:

a) Newton's 2nd law on each block and then add the equations.

Label the contact forces: F_{AB}= force of B on A (or A on B), and F_{BC}= force of C on B (or B on C). Take right as positive.

Block A (mass m_A), horizontal forces: applied F to the right and contact F_{AB} to the left:

$$F - F_{AB} = m_A a. \qquad (1)$$

Block B (mass m_B), horizontal forces: F_{AB} to the right from A and F_{BC} to the left from C:

$$F_{AB} - F_{BC} = m_B a. \qquad (2)$$

Block C (mass m_C), horizontal force: F_{BC} to the right:

$$F_{BC} = m_C a. \qquad (3)$$

Now add (1) +(2) +(3):

$$(F - F_{AB}) + (F_{AB} - F_{BC}) + (F_{BC}) = m_A a + m_B a + m_C a.$$

On the left the internal contact forces cancel: $-F_{AB} + F_{AB} - F_{BC} + F_{BC} = 0$. So, we get: $F = (m_A + m_B + m_C)\, a.$

Solve for a:

$$a = \frac{F}{m_A + m_B + m_C} = \frac{20}{2.0 + 7.0 + 4.0} = \frac{20}{13} = 1.54\ \text{m/s}^2 \approx 1.5\ \text{m/s}^2$$

b) Net force on each block

$$F_{\text{net},A} = m_A a = 2.0 \times 1.54 = 3.08\ \text{N} \approx 3.1N$$
$$F_{\text{net},B} = m_B a = 7.0 \times 1.54 = 10.78\ \text{N} \approx 11\text{N}$$
$$F_{\text{net},C} = m_C a = 4.0 \times 1.54 = 6.16\ \text{N} \approx 6.2N$$

From (1) $F - F_{AB} = m_A a$

c)

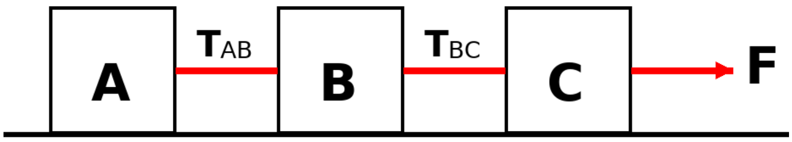
$$F_{AB} = N_{AB} = (20 - 2 \times 1.54) = 16.9\ \text{N}$$

d)

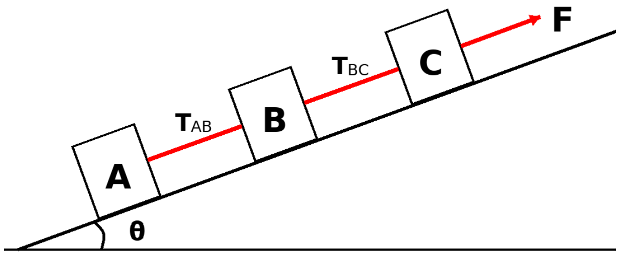
$$N_{BC} = m_C a = 6.2\ \text{N}$$

Exercise 2: Study the following systems, write equations of applying Newton's second law for each object, and add the equations to get the equation of acceleration of the system.

1) masses connected by ropes' mass is zero (or negligible), friction is negligible, A constant horizontal force $\boldsymbol{F}$ is applied.



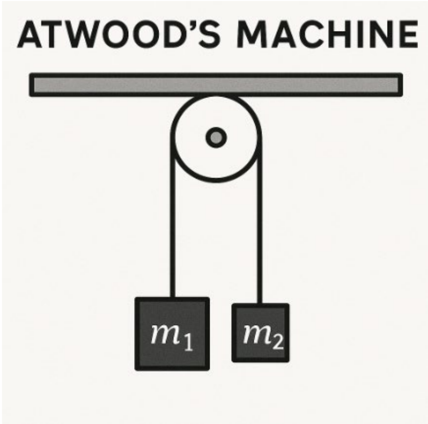
2) masses connected by ropes, ropes' mass is zero (or negligible), friction is negligible, the masses are pulled with a constant force $\boldsymbol{T}$



3.5.2 Smooth pulleys

A pulley is a wheel that rotates about a fixed axis, around which a rope or cord is wrapped. It is mainly used to change the direction of a force. When the pulley is smooth, the tension in the rope on both sides is equal in magnitude, since there is no energy loss due to friction or any conversion in the type of motion (for example, from linear to rotational).

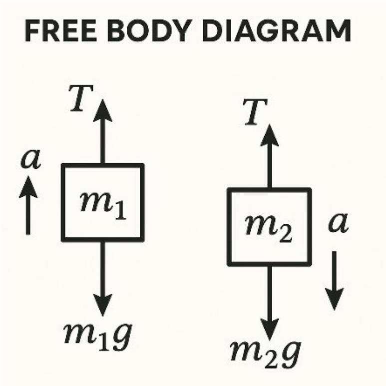
Atwood machine: Two masses joined together by a light rope passing over a fixed, smooth, negligible mass pulley. The device is sometimes used in the laboratory to measure the free fall acceleration.



Example 3-6

Two masses m_1 and m_2 are connected by a light, inextensible string that passes over a frictionless, massless pulley, forming an Atwood's machine. Derive expressions for (a) the acceleration of the system and (b) the tension in the string.

- Solution



- Step 1: Analyze the forces

Each mass experiences two forces:

- Its weight (mg) acts vertically downward.
- The tension (T) in the string, directed upward.

Assume $m_2 > m_1$ so that m_2 moves downward and m_1 moves upward with the same magnitude of acceleration a.

- Step 2: Apply Newton's Second Law (net force = mass × acceleration)

For m_2:

$$m_2g - T = m_2a \qquad (1)$$

For m_1:

$$T - m_1 g = m_1 a \qquad (2)$$

- Step 3: Eliminate the tension T

Add equations (1) and (2):

$$(m_2 - m_1)g = (m_1 + m_2)a$$

Solve for a:

$$\boxed{a = \frac{(m_2 - m_1)g}{m_1 + m_2}}$$

Direction: The heavier mass (m_2) accelerates downward, and the lighter mass (m_1) accelerates upward.

- Step 4: Find the tension T

Substitute the expression for $a$into either equation (1) or (2).

Using equation (2):

$$T = m_1(g + a)$$

After substitution:

$$\boxed{T = \frac{2m_1 m_2}{m_1 + m_2} g}$$

Check

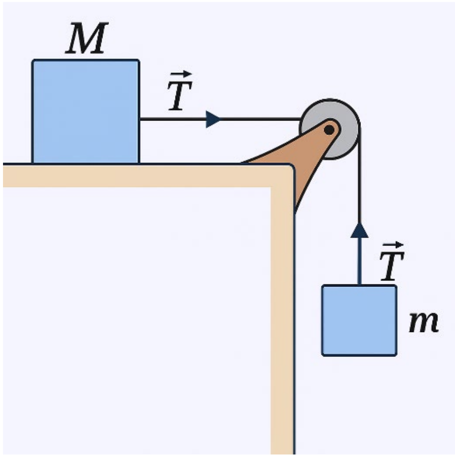
- If $m_1 = m_2$, then $a = 0$ (system in equilibrium).
- If $m_2 \gg m_1$, then $a \approx g$ (the heavier mass nearly falls freely).

Exercise 3: A block of mass M rests on a smooth horizontal table and is connected by a light, inextensible string that passes over a frictionless pulley to a hanging mass m, as shown in the figure.

Find:

(a) the acceleration of the system,

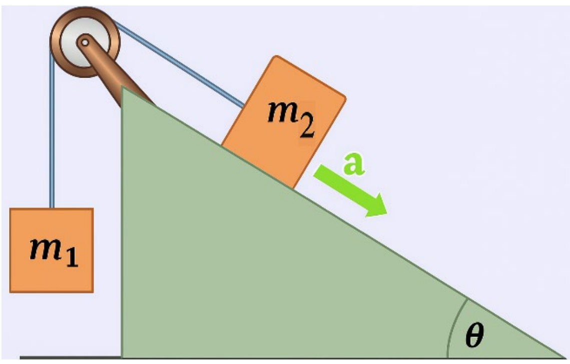
(b) the tension in the string.



Exercise 4: Two blocks are attached by a lightweight cord that passes over a frictionless pulley of negligible mass, as in fig. Find:

(a) the acceleration of the system,

(b) the tension in the string.



3.6 Forces of Friction

We have neglected friction up to this point, even though taking it into account is necessary in most practical situations. Friction force originates primarily from electrostatic forces between contacting surfaces (the nature of electrostatic forces in solid materials is still not completely understood), or from collisions between the rough microscopic bumps on the two surfaces.

The direction of the friction force is parallel to the surfaces in contact and opposite to the direction of motion, or attempted relative motion, between them. To understand what happens in static and kinetic friction, let us consider the case of an object starting to move from rest:

We apply a force to move the object. The object does not move because of static friction. We increase the applied force, and the static friction force increases accordingly until it reaches its maximum value $f_{s\,\max}$. If we increase the applied force beyond this point, the object begins to move, and kinetic friction appears—whose magnitude is less than the maximum static friction force.

3.6.1 Force of static friction

maximum Force of static friction:

$$f_{s\,max} = \mu_s F_N$$

μ_s: coefficient of static friction

3.6.2 Force of kinetic friction

Force of kinetic friction:

$$f_k = \mu_k F_N$$

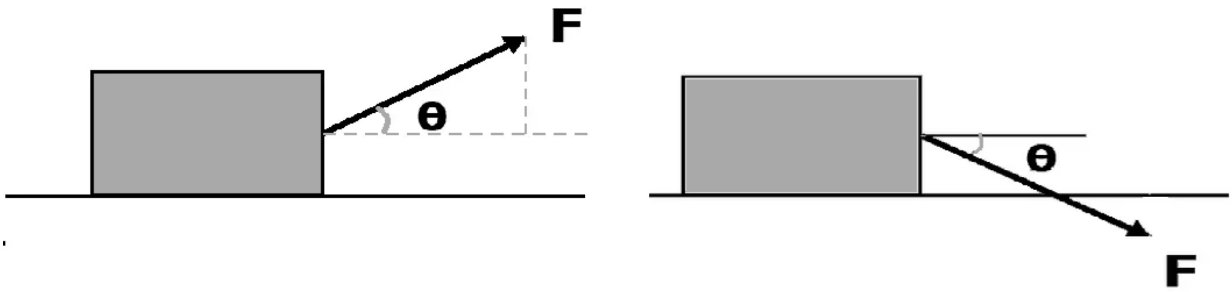
μ_k : coefficient of kinetic friction

They are both without units, and their values are usually less than one. $\mu_k < \mu_s$

Important Notes

coefficient of static friction μ_k is roughly independent of the sliding speed, as well as the area in contact.

Think: If you wanted to move an object, which of the two directions in the diagram would be easier to do with the same magnitude of force and the same θ?



Materials in Contact	Static Friction (μ_s)	Kinetic Friction (μ_k)
Steel on steel	0.74	0.57
Aluminum on steel	0.61	0.47
Copper on steel	0.53	0.36
Rubber on concrete	1.00	0.80
Wood on wood	0.25–0.50	0.20
Glass on glass	0.94	0.40
Waxed wood on wet snow	0.14	0.10
Waxed wood on dry snow	—	0.04
Metal on metal (lubricated)	0.15	0.06
Ice on ice	0.10	0.03
Teflon on Teflon	0.04	0.04
Synovial joints in humans	0.01	0.003

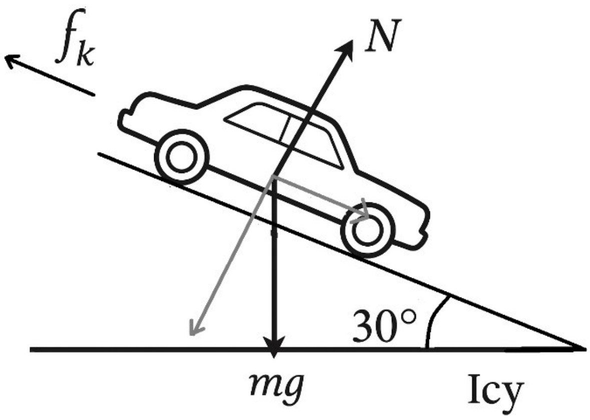
Example 3-7

A broken-down car of mass $1.0 \times 10^3\ kg$ is on an icy driveway inclined at an angle $30.0°$

(a) Find the acceleration of the car, if the coefficient of kinetic friction is 0.10

(b) Suppose the car slides from rest at the top of the incline, and the distance from the car's front bumper to the bottom of the incline is $d = 1.00 \times 10^2\ m$. How long does it take the front bumper to reach the bottom, and what is the car's speed as it arrives there?

Solution:



Given: $m = 1.0 \times 10^3$ kg, $\theta = 30.0°$, $\mu_k = 0.10$, $g = 9.80$ m/s^2.

(A) Acceleration

Normal: $N = mg\cos\ \theta$.

Kinetic friction (up the slope): $f_k = \mu_k N = \mu_k mg\cos\ \theta$.

Along the slope:

$\Sigma F = mg\sin\ \theta - f_k = mg\sin\ \theta - \mu_k mg\cos\ \theta$

$$a = \frac{\Sigma F}{m} = \frac{\text{mgsin}\,\theta - \mu_k\,\text{mgcos}\,\theta}{m} = g(\sin\theta - \mu_k\cos\theta) = \left(9.80\ \text{m/s}^2\right)(0.500 - 0.10 \times 0.866)$$

$$a = 4.05\ m/s^2 \approx 4.0\ m/s^2$$

(B) From rest, distance $d = 1.00 \times 10^2$ m $= 100$ m

$$d = \frac{1}{2}at^2$$

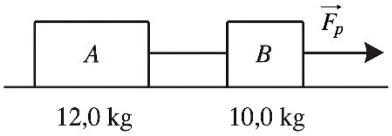
$$t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{2(100\ \text{m})}{4.05\ \text{m/s}^2}} \approx \boxed{7.0\ \text{s}}$$

Speed at the bottom:

$$v = \sqrt{2ad} = \sqrt{2(4.05\ \text{m/s}^2)(100\ \text{m})} \approx 28.5\ m/s$$

Note that we did not need the mass of the car in the solution.

Exercise 5: Two boxes connected by a cord. Two boxes, A and B, are connected by a lightweight cord and are resting on a smooth (frictionless) table. The boxes have masses of $12.0\ kg$ and $10.0\ kg$. A horizontal force F_p of $40.0\ N$ is applied to the $10.0kg$ box, as shown in Fig. Find (a) the acceleration of each box, and (b) the tension in the cord connecting the boxes.



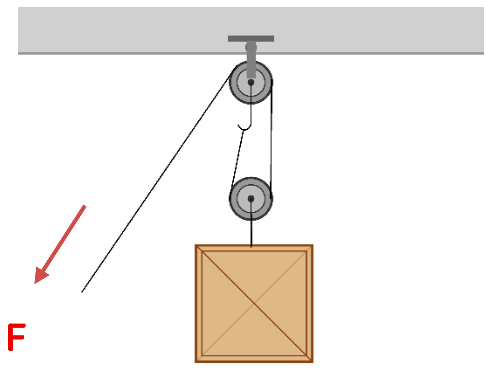
Exercise 6: A 10.0 kg g box is pulled along a horizontal surface by a force F_p of 40.0 N applied at a 30.0° angle above horizontal., and we assume a coefficient of kinetic friction of 0.30. Calculate the acceleration.

Exercise 7: Suppose a block is placed on a rough surface inclined relative to the horizontal. The incline angle is increased until the block starts to move. Show that by measuring the critical angle θ_c at which this slipping just occurs, we can obtain μ_s .

3.7 Additional problems

1- A 65-kg woman descends in an elevator that briefly accelerates at $0.20g$ downward. She stands on a scale that reads in kg. (a) During this acceleration, what is her weight and what does the scale read? (b) What does the scale read when the elevator descends at a constant speed of $2.0\ m/s$?

2- A person is trying to lift a crate upward at a constant speed and slowly, using a rope that passes around two pulleys: the upper pulley is fixed, and the lower pulley can move up and down as shown in the figure. What is the magnitude of the force F that the person must apply to the rope if the



weight of the crate is 800 N?

3- The skier has begun descending the $30.0°$ slope. If the coefficient of kinetic friction is 0.10, find:

(a) Her acceleration.

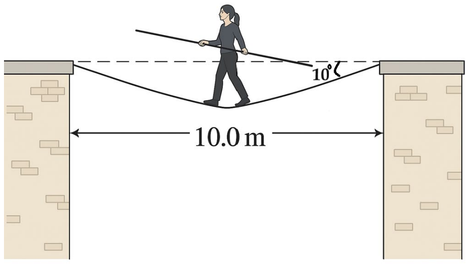
(b) The speed that will be reached after 4.0 s.

4- A $20.0\ kg$ box rests on a table. (a) What is the weight of the box and the normal force acting on it? (b) A $10.0\ kg$ box is placed on top of the $20.0\ kg$ box, determine the normal force that the table exerts on the $20.0\ kg$ box and the normal force that the $20.0\ kg$ box exerts on the $10.0\ kg$ box.

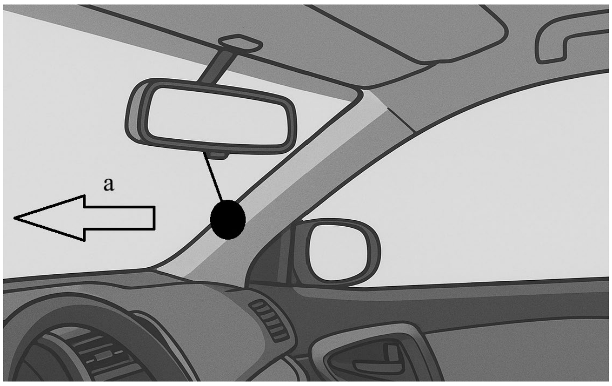
5- What average force is required to stop a $1100\ kg$ car in $8.0s$ if the car is traveling at $95\ km/h$?

6- A person stands on a bathroom scale in a motionless elevator. When the elevator begins to move, the scale briefly reads only 0.75 of her regular weight. Calculate the acceleration of the elevator and find the direction of acceleration.

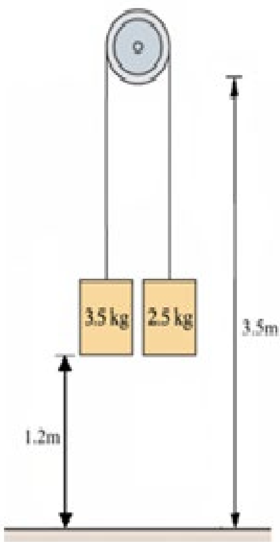
7- Layla is to walk across a "high wire" strung horizontally between two buildings $10.0\ m$ apart. The sag in the rope when she is at the midpoint is $10.0°$, as shown in Fig. If her mass is $50.0\ kg$, what is the tension in the rope at this point?



8- A ball is suspended by a string hanging from a car's rear-view mirror. What angle will the string make with the vertical while the car accelerates from rest at a traffic light to a speed of $18.0\ m/s$ in $6.0\ s$?

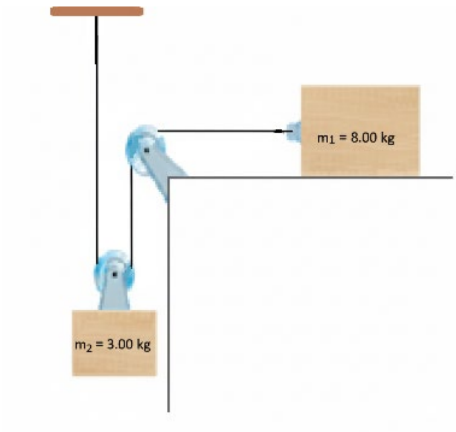


9- When the system in the figure is released, what is the maximum height the mass $2.5\ kg$ will reach, neglecting air resistance?



10- A person jumps from the roof of a house $3.9m$ high. When he strikes the ground below, he bends his knees so that his torso decelerates over an approximate distance of $0.70\ m$. If the mass of his torso (excluding legs) is $42\ kg$, find (a) his velocity just before his feet strike the ground, and (b) the average force exerted on his torso by his legs during deceleration.

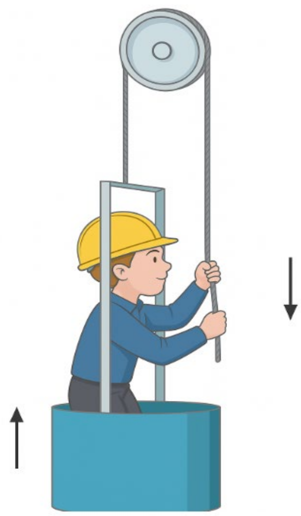
11- In the drawing, the rope and the pulleys are massless, and there is no friction. Find (a) the tension in the rope and (b) the acceleration of each block.



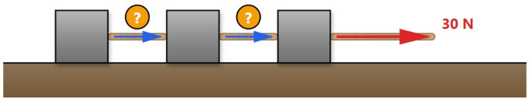
12- A $7.00 \times 10^2\ kg$ window-cleaning worker pulls himself upward using a machine (consisting of a pulley and a bucket), as shown in the figure.

(a) What force must he pull downward with to rise slowly at a constant speed?

(b) What is his acceleration if he increases the pulling force by 20%? Assume the total weight of the worker and bucket is $7.00 \times 10^2\ kg$.

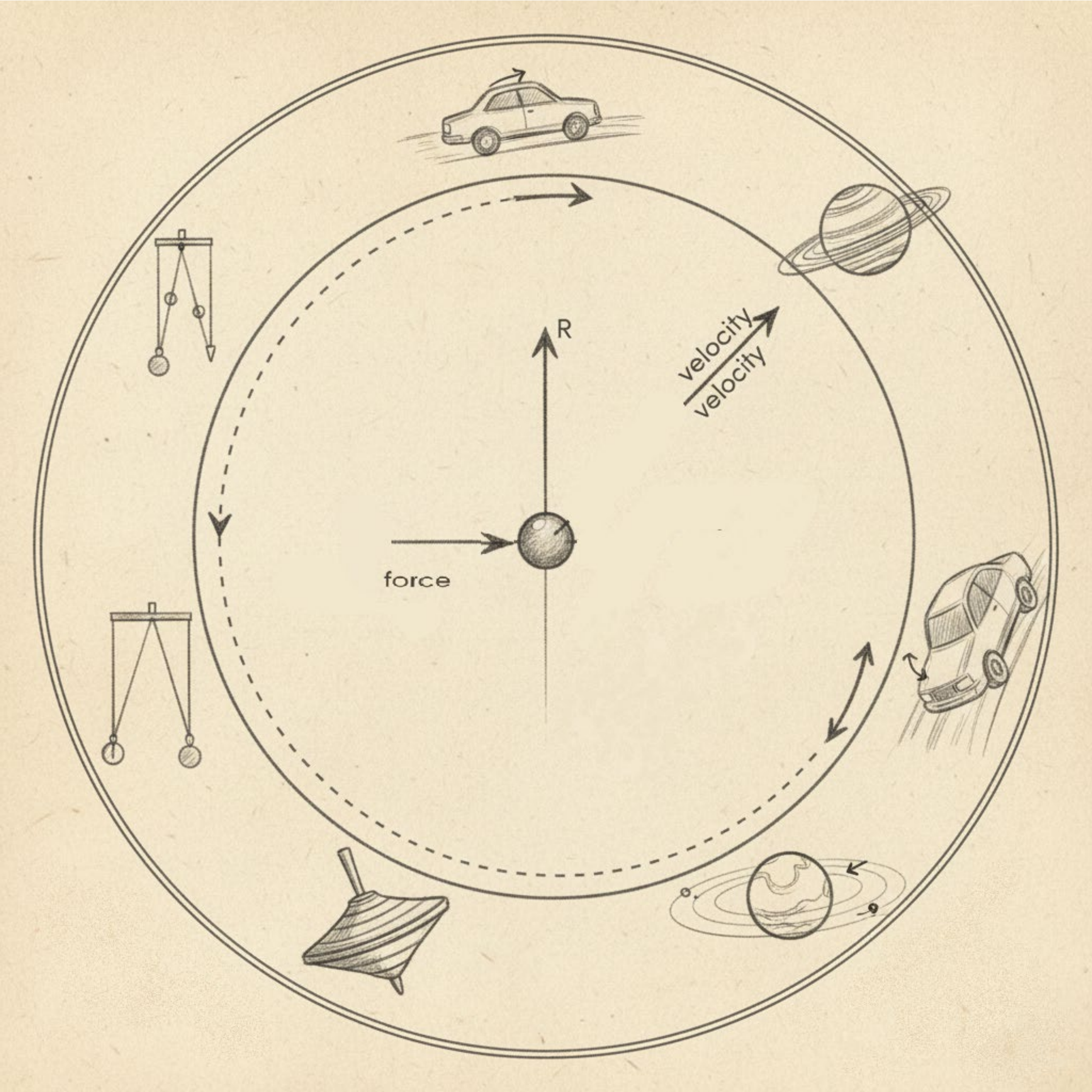


13- Three identical cubes are pulled as shown in the figure on a horizontal surface without friction. If the tension in the rope held by the hand is $30.0\ N$, what is the value of the tension in the other ropes?



14- A motorcycle and a $60.0-kg$ rider accelerate at $3.0 m/s^2$ up a ramp inclined $10.0°$ above the horizontal .What is the magnitude of (a) the net force on the rider, and (b) the force exerted on the rider by the motorcycle?

4 Circular Motion



4.1 Uniform Circular Motion

We observe many circular motions in our daily lives, such as the movement of a car on a circular curve, the motion of the moon, a Ferris wheel, and others. These are examples of two-dimensional motion that we have learned about previously.

Motion of an object on a circular path of radius $r$with a tangential velocity v :constant of value and variable of direction.

4.1.1 Quantities of Uniform Circular Motion:

Tangential velocity (v):

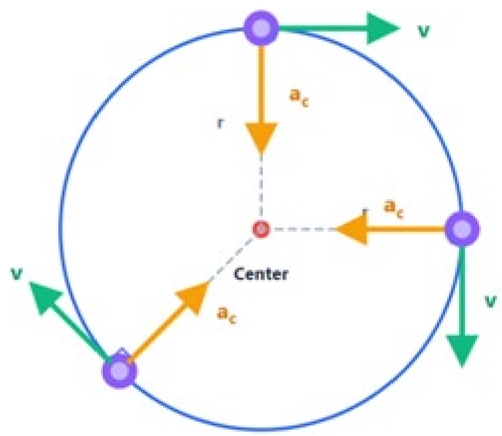
It is always perpendicular to the radius of rotation.

Centripetal (or radial) acceleration (a_c):

It is always directed toward the center of the circle and is responsible for changing the direction of the velocity.

Centripetal force (F_c):

It is always directed toward the center of the circle and is responsible for producing the acceleration.



Periodic Time T:

The time required to complete one full revolution

It is measured in seconds (s) in the International System of Units (SI).

Frequency f:

The number of revolutions per unit time

It is measured in hertz (Hz), where one hertz equals one revolution per second:

$$\boldsymbol{Hz = rps}$$

Laws of Uniform Circular Motion

$$\boldsymbol{a_c = \frac{v^2}{r}}$$

$$\boldsymbol{F_c = ma_c = m\frac{v^2}{r}}$$

$$\boldsymbol{f = \frac{1}{T} = \frac{v}{2\pi r}}$$

$$\boldsymbol{T = \frac{2\pi r}{v}}$$

$$\boldsymbol{v = 2\pi r f}$$

Check concept: An object moves at constant speed along a circular path in a horizontal $\boldsymbol{xy}$ plane, with the center at the origin. When the object is at $\boldsymbol{x = -2m}$, its velocity is $\boldsymbol{(-4\ m/s)\ j}$. Give the object's (a) velocity (b) acceleration at $\boldsymbol{y = 2\ m}$.

Example 4-1

The Moon's orbit around the Earth is roughly circular. With an average radius $\boldsymbol{3.84 \times 10^8 m}$, the Moon takes $\boldsymbol{27.3}$ days to complete a complete revolution around the Earth. Calculate: (a) The average orbital velocity of the Moon. (b) Its perpendicular acceleration.

Solution:

Given Data:

$$Orbital\ radius, r\ =\ 3.84\ \times\ 10^8\ m$$

$$Orbital\ period, T\ =\ 27.3\ days\ =\ 27.3\ \times\ 24\ \times\ 3600\ =\ 2.35872\ \times\ 10^6\ s$$

$(a)\ Average\ Orbital\ Velocity$

$$v\ =\ 2\pi r\ /\ T$$

$$v\ =\ (2\ \times\ \pi\ \times\ 3.84\ \times\ 10^8)\ /\ (2.35872\ \times\ 10^6)$$

$$v = 1.023 \times 10^3\, m/s \approx 1.02\, km/s$$

(b) Perpendicular (Centripetal) Acceleration

$$a_c = v^2 / r$$
$$a_c = (1.023 \times 10^3)^2 / (3.84 \times 10^8)$$
$$a_c = 2.72 \times 10^{-3}\, m/s^2$$

Example 4-2

Identify the source of the centripetal force in each of the following cases:

(a) A car moving in a circular path around a roundabout.

(b) A stone tied to a string and swung horizontally (approximately parallel to the ground).

(c) Satellites orbiting the Earth.

Solution:

(a) The static friction force between the tires and the road surface.

(b) The tension force in the string.

(c) The gravitational force (mutual gravitational attraction).

Exercise 1: A tire 0.500 m in radius rotates at a constant rate of $2.00 \times 10^2\ rev/min.$ Find the speed and acceleration of a small stone lodged in the tread of the tire (on its outer edge).

Exercise 2: Estimate the force a person must exert on a string attached to a $0.150\ kg$ ball to make the ball revolve in a horizontal circle of radius 0.600 m. The ball makes 2.00 revolutions per second. Ignore the string's mass.

4.2 Vertical Circular Motion

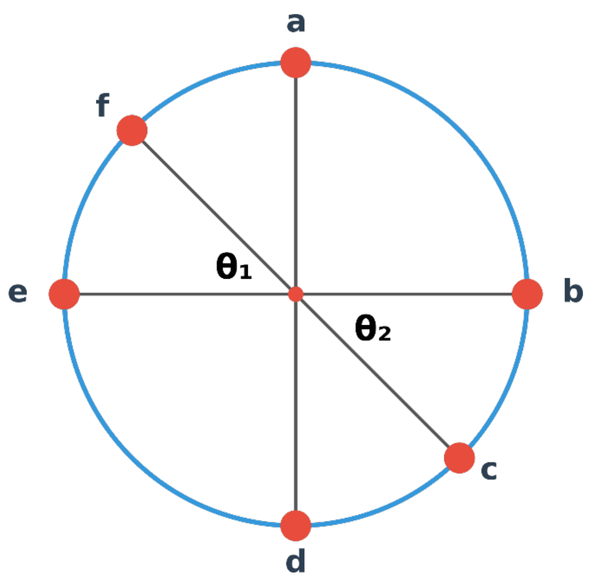
It is the motion of an object in a vertical circular path, such as the motion of an object tied to a string, where a person rotates it with tension force $\boldsymbol{T}$. The centripetal force is the resultant of forces in the radial direction (i.e., the weight and the tension force from the person), and therefore the centripetal force takes different values at different points. Remember that the centripetal force at any position is related to the object's speed by the relation

$$F_c = m\frac{v^2}{r}$$

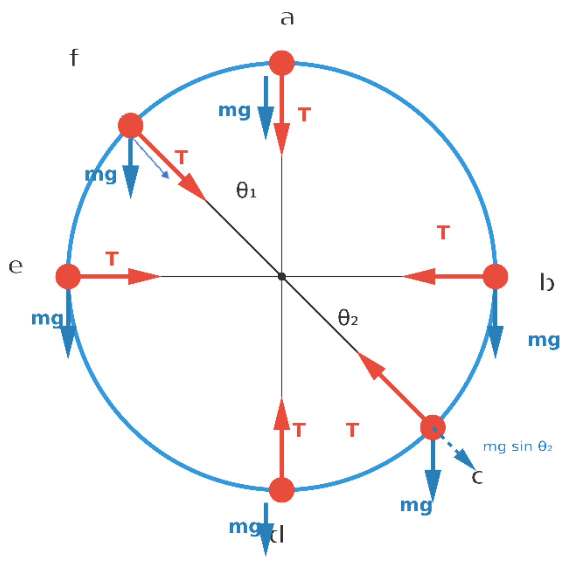
Note: The centripetal force is not a separate force, but rather the net force in the radial direction.

Example 4-3

The figure shows the positions of a small ball attached to a light (massless) string, rotating in a vertical circular path. Write the centripetal force F_c in terms of the weight (or its components) and the tension at each position.



Solution: We calculate the magnitude of F_c for each position, and its direction is always towards the center of rotation.



(a) $Fc = T + mg$
(b) $Fc = T$
(c) $Fc = T - mg \sin \theta_2$
(d) $Fc = T - mg$
(e) $Fc = T$
(f) $Fc = T + mg \cos \theta_1$

Example 4-4:

Find the minimum speed of an object moving in a vertical circular path.

Solution:

At the highest point, the minimum speed required for circular motion occurs when:

$$F_c = \frac{mv^2}{r} = mg$$
$$v_{min} = \sqrt{rg}$$

Concept Check:

What happens if the speed is less than $\sqrt{rg}$ for an object moving in a vertical circular path?

Concept Check:

Does the weight of the object affect the minimum speed of an object moving in a vertical circular path?

Exercise 3: A small ball of mass $0.200\ kg$ is attached to a $1.25\ m$ long light string and is whirled in a vertical circle.

(a) Determine the minimum speed the ball must have at the highest point of its path to maintain a taut string (i.e., so the ball continues in circular motion).

(b) If the ball is moving at three times the minimum speed found in part (a) when at the bottom of the circle, calculate the tension in the string at the bottom of the motion.

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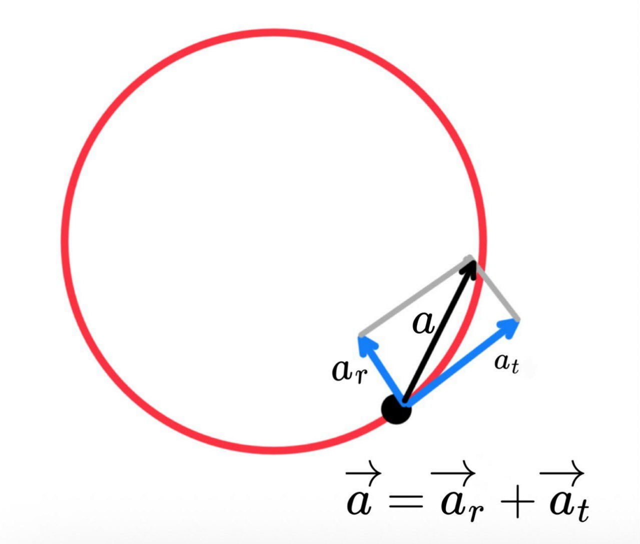
4.3 Nonuniform Circular Motion



In non-uniform circular motion, the magnitude of tangential velocity changes as its direction changes, and in this case, the two perpendicular components of acceleration are the tangential component $\boldsymbol{a_t}$ and the perpendicular (radial) component $\boldsymbol{a_r}$.

The tangential component $\boldsymbol{a_t}$: Changes the value of velocity of an object and it is parallel to it.

When $\boldsymbol{a_t}$ is at in a direction of $\boldsymbol{v}$? and when is it in reverse direction?

The radial component $\boldsymbol{a_r}$: Changes the direction of velocity of an object and it is perpendicular to it.

The total acceleration

$$\boldsymbol{a} = \boldsymbol{a_t} + \boldsymbol{a_r}$$

$$\boldsymbol{a} = \sqrt{\boldsymbol{a_t}^2 + \boldsymbol{a_r}^2}$$

$$\boldsymbol{a_r} = \frac{\boldsymbol{v}^2}{\boldsymbol{r}}$$

r: The radius of the path curve at the desired point

Consequently, two forces act on the body: a tangential force that produces the tangential acceleration, and a centripetal force that produces the radial (centripetal) acceleration.

Think: When circular motion is uniform, what is the value of F_t?

Example 4-5

A motorcycle travels over the top of a small hill. The hill can be approximated as the arc of a circle with a radius of $350\ m$. At the instant the motorcycle reaches the highest point of the hill, its speed is $8.0\ m/s$. At the same moment, the rider is accelerating forward along the road with a tangential acceleration of $0.450\ m/s^2$.

(a) Determine the magnitude of the normal (centripetal) component of the acceleration at this point.

(b) Find the magnitude of the total acceleration.

(c) State the direction of the total acceleration relative to the horizontal (give the angle below the horizontal).

Solution:

Given:

- Radius of the hill: r = 350 m
- Speed at the top: v = 8.0 m/s
- Tangential acceleration: a_t = 0.450 m/s²

(a) Calculate the normal (centripetal) acceleration:

The centripetal acceleration at the top is calculated using:

$$a_n = v^2/r$$

$$a_n = (8.0)^2 / 350$$

$$a_n = 0.183\ m/s^2$$

(This acceleration is directed downward toward the center of the circle)

(b) Calculate the magnitude of the total acceleration:

The total acceleration is the vector sum of the tangential and normal accelerations:

$$a = \sqrt{(a_t{}^2 + a_n{}^2)}$$

$$a = \sqrt{[(0.450)^2 + (0.183)^2]}$$

$$a = 0.486\ m/s^2$$

(c) Direction of the total acceleration:

The angle below the horizontal (θ) is calculated from:

$$\theta = tan^{-1}\,(\frac{a_r}{a_t})$$

$$\theta = tan^{-1}\,(\frac{0.183}{0.450})$$

$$\theta = 22.1°$$

Exercise 4: A small ball attached to a string of length $0.80m$ swings in a vertical circle. When the string makes an angle $\theta = 35°$with the vertical, the speed of the ball is 2.2 m/s

(a) Find the normal (centripetal) component of acceleration.

(b) Find the tangential acceleration at that instant.

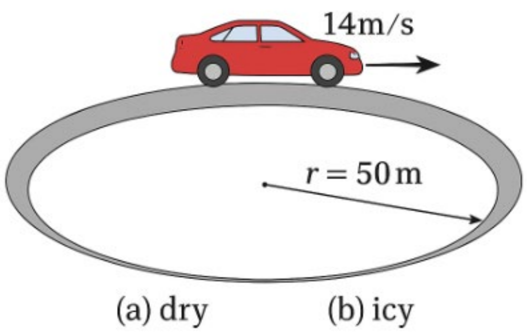
(c) Determine the magnitude of the total acceleration and the angle it makes relative to the inward radial direction.

4.4 Additional problems

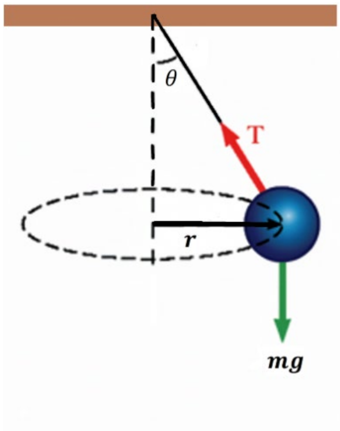
1- An astronaut on the surface of moon fires a cannon to launch an experiment package, which leaves the barrel moving horizontally. (a) What must be the muzzle speed of the package so that it travels completely around the moon and returns to its original location? (b) How long does this trip around the Moon take? Assume that the free fall acceleration on the moon is one-sixth that on the Earth and moon radius is $1740\ km$

2- What is the magnitude of the Centripetal acceleration, of a pilot whose aircraft enters a horizontal circular turn with a velocity of: $v_i = (400i + 500j)\ m/s$ and $24.0\ s$ later leaves the turn with a velocity of $v_f = (-400i - 500j)\ m/s.$

3- A $1.0 \times 10^3\ kg$ car rounds a curve on a flat road of $5.0 \times 10^1\ m$ radius at a speed of $14\ m/s$. Will the car follow the curve, or will it skid? Assume: (a) the pavement is dry, and the coefficient of static friction is $\mu_s = 0.60$ (b) the pavement is icy and $\mu_s = 0.25$

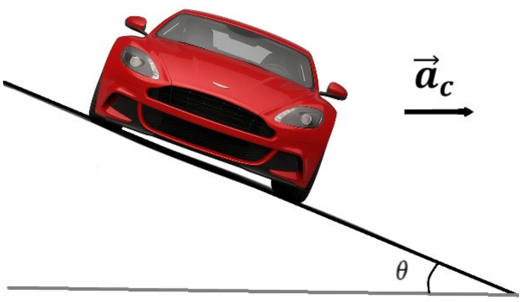


4- A small object of mass m is suspended by a string of length L and revolves at a constant speed in a horizontal circle of radius r. Find v in terms of L, g, and the sine and tangent of the angle θ.



5- (a) What is the angle θ of banking for a road curve such that no friction is required for a car traveling at speed $v$around a curve of radius r? Find θ in terms of v, r,and g.

(b) Calculate the banking angle of a curved road with a radius of 70.0 m that is designed for safe driving at a speed of 60.0 km/h.



6- A stunt pilot of mass m flies an aircraft in a vertical loop maneuver. The aircraft travels in a circular path of radius 2.10 km at a constant speed of 240 m/s. Determine the force exerted by the seat on the pilot at:

(a) The lowest point of the loop.
(b) The highest point of the loop.

7- A high-speed tram enters a curved section of track and slows down from $72.0\ km/h$ to $40.0\ km/h$ in the $12.0\ s$ it takes to travel through the curve. The radius of the track's curvature is $200\ m$.

(a) Determine the tangential acceleration of the tram during this interval, assuming the rate of slowing is constant.
(b) At the instant the tram reaches 40.0 km/h, compute the centripetal acceleration.
(c) Determine the magnitude of the total acceleration of the tram at this same instant.

THE PRACTICE TEST – SECOND STAGE

1) The position of a moving object is given by the equation: $r = \beta t^3 + \alpha$. Where t is time. What are the dimensions of β and α respectively?

A) $[\beta] = [\mathrm{L\,T^{-3}}], [\alpha] = [\mathrm{L}]$

B) $[\beta] = [\mathrm{L\,T^{-2}}], [\alpha] = [\mathrm{L}]$

C) $[\beta] = [\mathrm{L\,T^{-3}}], [\alpha] = [\mathrm{T}]$

D) $[\beta] = [\mathrm{T^{-3}}], [\alpha] = [\mathrm{L\,T^{-1}}]$

---

2) The relationship between the energy of a particle E and its temperature T is given by: $E = k_B T$. where E is the energy with dimensions $[M L^2 T^{-2}]$, and T is the temperature in kelvins. What are the dimensions of the Boltzmann constant k_B ?

A) $[M L T^{-2}]$

B) $[M L^2 T^{-2}]$

C) $[M L^2 T^{-2} K]$

D) $[M L^2 T^{-2} K^{-1}]$

---

3) Assume that the mass of the largest particle that can be carried by the air depends only on the air velocity v, the air density ρ, and the acceleration due to gravity g .

Which of the following relationships shows the proportion between the maximum mass that air can hold (M) and each of the following: ρ, v, and g ?

A) $M \propto \rho v^3 / g^2$

B) $M \propto \rho v^4 / g^2$

C) $M \propto \rho v^6 / g^3$

D) $M \propto \rho v^5 / g^2$

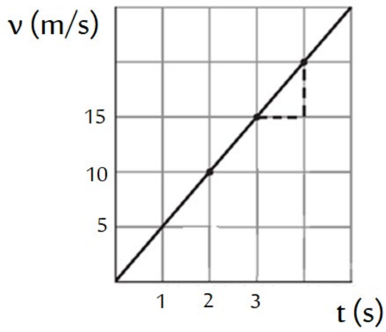
4) A nut falls from the ceiling of a train car. The train is moving toward the north, and its speed is increasing at a rate of 3.20 m/s^2 (the nut is inside the train car). What is the acceleration of the nut relative to an observer sitting in the same car, assuming that the positive directions are toward the north and upward?

A) $(0\,\hat{i} - 9.80\,\hat{j})\ m/s^2$
B) $(3.20\,\hat{i} - 9.80\,\hat{j})\ m/s^2$
C) $(-3.20\,\hat{i} - 9.80\,\hat{j})\ m/s^2$
D) $(-9.80\,\hat{i} - 3.20\,\hat{j})\ m/s^2$

--

5) A 5 kg object moves along a rough surface in a straight line, and its speed changes as shown in the diagram. If the object is moving under the influence of a horizontal tension force of 30 N, then the magnitude of the frictional force is:

A) 25N
B) 10N
C) 5N
D) 3N



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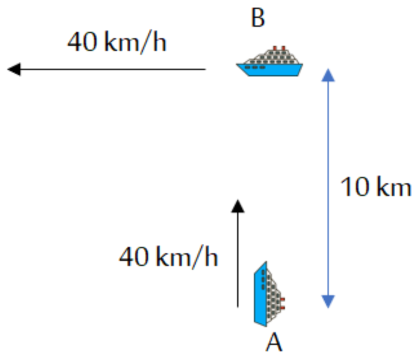
6) Two ships, A and B, are 10.0 km apart on a south-north line, as shown in the figure. Ship B begins moving west at a constant speed of 40.0 km/h, and Ship A begins moving north at a constant speed of 40.0 km/h. The closest distance between them during their motion will be:
Hint: You can start by calculating the speed of ship A relative to ship B.

A) 5.00 km
B) 6.40 km
C) 7.07 km
D) 10.0 km



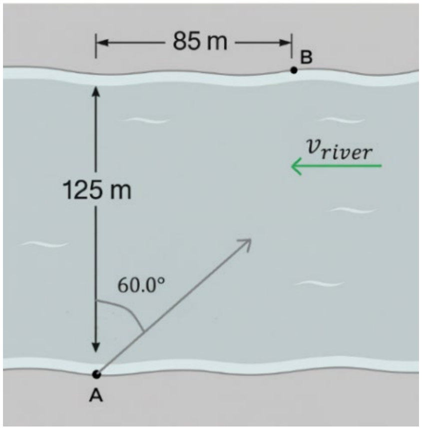
7) A boat has a speed of 0.8 m/s in still water. The swimmer heads his boat at an angle of 60^0 from the vertical to cross a river that is 125 m wide, moving from point A to point B in a straight line, as shown in the figure. The speed of the river's current is equal to:

A) 0.96 m/s

B) 0.69 m/s

C) 0.40 m/s

D) 0.42 m/s



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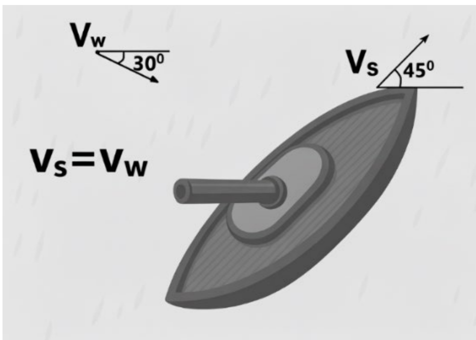
8) A ship moves at constant speed v_s relative to the earth, The wind is blowing at a speed of v_w relative to the earth as shown in the figure. Which vector represents the speed of the ship's smoke as seen from an observer on the ship?

Assume the smoke has a negligible upwards velocity. In other words, it travels parallel to the surface of the water.



 (B

 (A

 (D

 (C

9) Which of the following facts correctly describes the concepts of mass and weight of an object?

A) The ratio between them at any point on Earth is 9.8

B) The ratio between them is equal to the ratio between them for another object placed in the same position.

C) These are two concepts that have the same meaning in physics but differ in their units.

D) They both express the magnitude of an object's inertia.

---

10) A net force F acts for 10 s on a body of mass 10^{-2} kg, initially at rest, after which the force ceases to act. The body traverses 0.5 m in the next 5 s with constant velocity. The magnitude of the force is:

A) 10^{-1} N

B) 10^{-2} N

C) 10^{-3} N

D) 10^{-4} N

---

11) A body of mass 2.00 kg is moving towards west with a uniform speed of 4.00 m/s. A force of 4.00 N is applied to it towards north. The magnitude of the displacement of the body 4.00 s after the force is applied is:

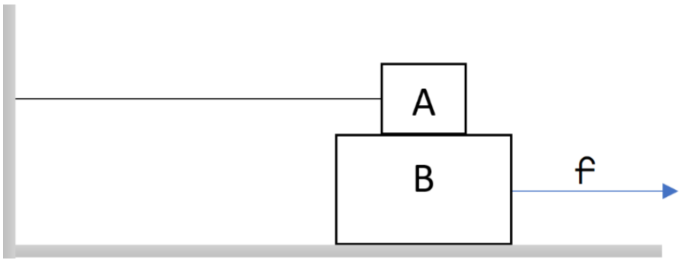
A) 5.66 m

B) 16.0 m

C) 22.6 m

D) 32.0 m

12) A block A of mass m_A rests on another block B of mass m_B . Block A is tied to a wall by a rope of negligible mass as shown in the figure, The coefficient of friction between A and B is 0.2 , that between B and the ground is 0.3 and the acceleration due to gravity is g. What is the minimum force (F) required to move block B?



A) $0.2\, m_A\, g + 0.3\, (m_A + m_B)g$

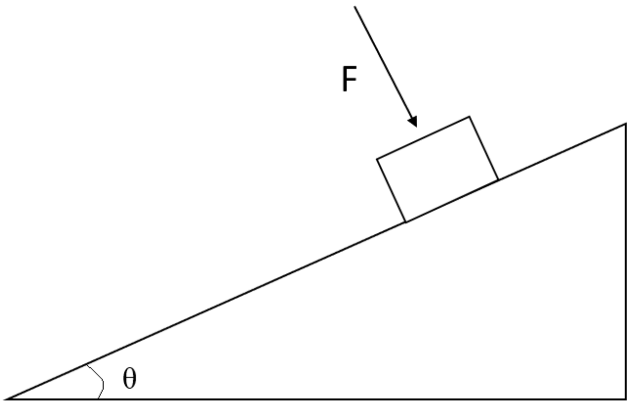
B) $0.2\, m_A\, g - 0.3\, (m_A + m_B)g$

C) $0.2\, m_A\, g + 0.3\, m_B\, g$

D) $(m_A + m_B)g$

--

13) A force F is used to keep a block of mass m stationery on a rough surface inclined with an angle θ to the horizontal. If the coefficient of static friction between the block and the inclined surface is μ, then the minimum value required for the force F is:



A) $\frac{mg(sin\theta - \mu cos\theta)}{\mu}$

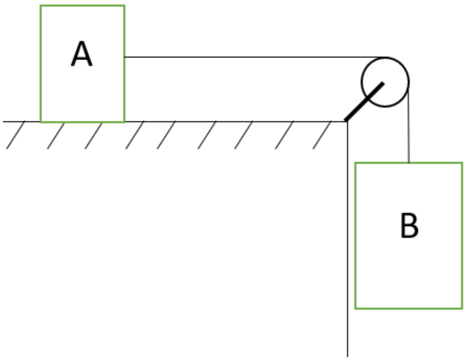
B) $\frac{mg(cos\theta - \mu sin\theta)}{\mu}$

C) $\frac{mg(sin\theta + \mu cos\theta)}{\mu}$

D) $\frac{mg(cos\theta + \mu sin\theta)}{\mu}$

14) Two blocks are attached by a light rope passing over a stationary, smooth pulley. The first block, with mass m_A, is placed on a rough horizontal plane with a coefficient of static friction μ_S. The second block is suspended, and its mass m_B .For the first block to start moving from rest, the value of the suspended block's mass must be: where g the acceleration due to gravity is:

A) $\mu_S m_A + 1$
B) $\mu_S m_A$
C) $g\mu_S m_A$
D) $\frac{\mu_S m_A}{g}$



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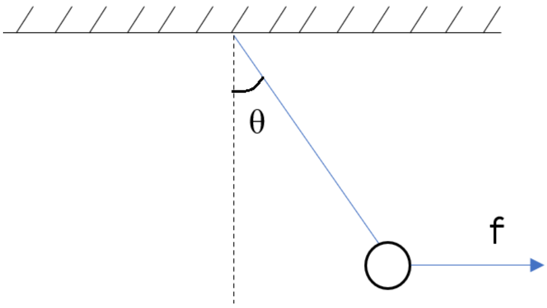
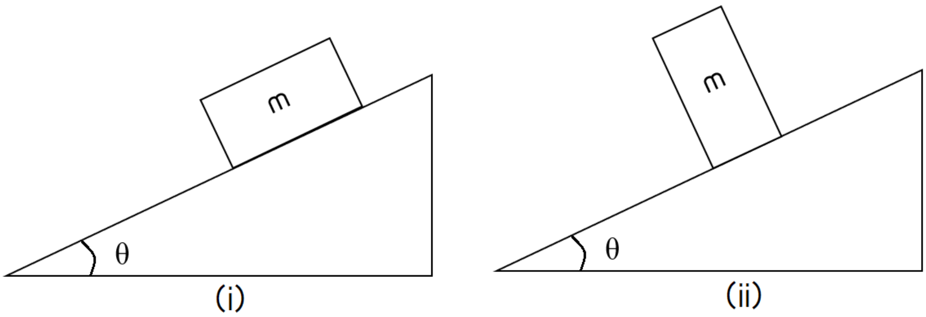
15) A pendulum of weight 1.0 N is fixed at an angle θ from the vertical by a horizontal force F = 2.0 N. The tension in the string supporting the pendulum (in Newtons) is equal to:

A) $cos\theta$
B) $2/cos\theta$
C) $\sqrt{5}$
D) 1



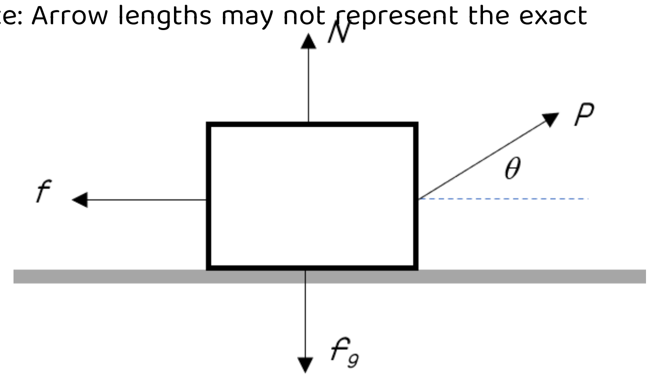
16) The same uniform block is first placed on its long side, then on its short side, on the same inclined plane, as shown in the figure. The block accelerates down the inclined plane (Assume the box will not tip over). The magnitude of the block's acceleration in case (ii) compared to its acceleration in case (i) is:



A) the same
B) greater
C) less
D) twice

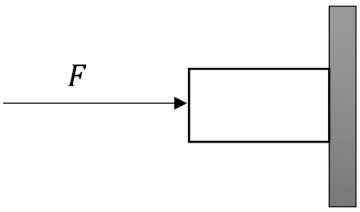
---

17) A boy pulls a wooden box along a rough horizontal surface at a constant speed using the force P, as shown in the figure. f is the frictional force, N is the normal force, and Fg is the gravitational force. Which of the following must be true? Note: Arrow lengths may not represent the exact values of forces.



A) P = F and N = F_g
B) P = F and N > F_g
C) P > F and N < F_g
D) P > F and N = F_g

18) A book of mass m is placed in contact with a wall and pushed with a horizontal force of magnitude F as shown in figure. The book slides downwards in contact with the wall. If the coefficient of static friction μ_s and the coefficient of kinetic friction μ_k, then the net force acting on it is equal to:



A) $mg - \mu_s F$
B) $mg - (\mu_{s-}\mu_k)F$
C) $mg - \mu_k F$
D) $mg - (\mu_{s+}\mu_k)F$

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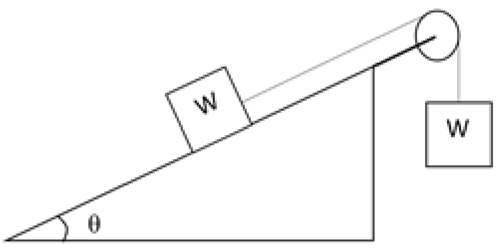
19) Two blocks of the same weight W are connected by a light rope passing over a fixed and smooth pulley as shown in the figure. If the blocks are at rest, the block on the inclined plane is subjected to a



A) W (1-sin θ)
B) W (1+sin θ)
C) W (1-cos θ)
D) W (1+cos θ)

20) A giant wheel with a diameter of 40.0 m is equipped with a cage and a platform on which a man of mass m stands. The wheel rotates in a vertical plane at a speed such that the force exerted by the man on the platform is equal to his weight when the cage is at point X. The man's speed at point X is:

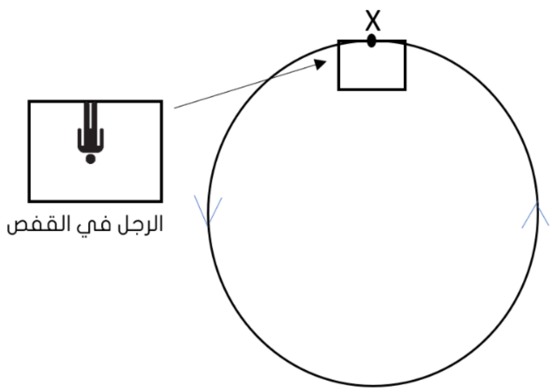
Consider: $g = 10.0\ m/s^2$

A) 14.0 m/s

B) 20.0 m/s

C) 28.0 m/s

D) 80.0 m/s



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21) A stone is moved in a horizontal circle with a radius of 1.50 m by a string suspended 2.00 m above the ground. The string breaks, and the stone flies horizontally, striking the ground 10.0 m horizontally from the point where it left the circular path. The centripetal acceleration during circular motion is: Consider $g = 10.0\ m/s^2$

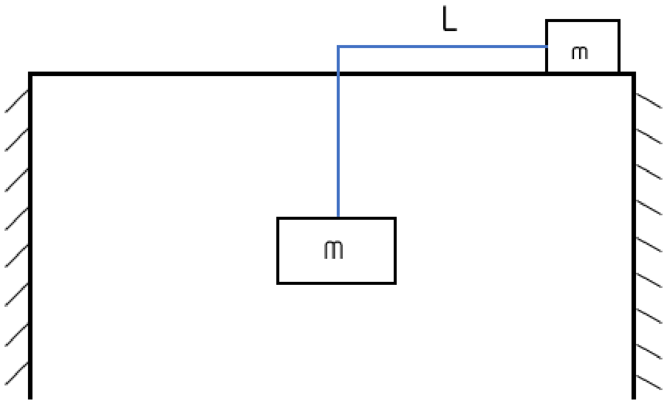
A) $10.5\ m/s^2$

B) $15.8\ m/s^2$

C) $112.2\ m/s^2$

D) $166.6\ m/s^2$

22) Two masses m and M are connected by a light string that passes through a smooth hole O at the
center of a table. Mass m lies on the table and M hangs vertically m is moved round in a horizontal circle with O as the center. If L is the length of the string from O to m, and all surfaces with neglected friction

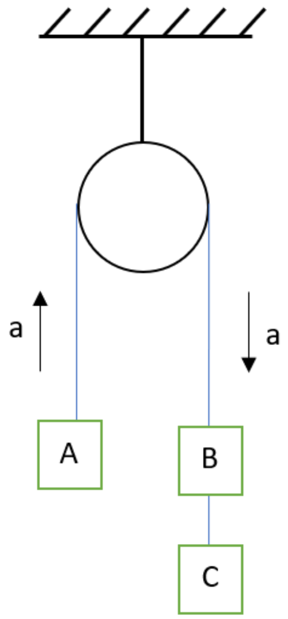


Then the rotation speed of mass m is equal to:

A	$\sqrt{\frac{m}{M}lg}$	C	$\sqrt{\frac{m}{lM}g}$
B	$\sqrt{\frac{M}{m}lg}$	D	$\sqrt{\frac{M}{lm}g}$

23) Three blocks A, B and C, each of mass 2 kg, are hanging over a fixed pulley and accelerate by a as shown in the figure. The tension in the string connecting B and C is:

A) A
B) 2a
C) 3a
D) 4a



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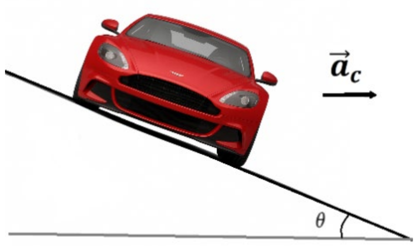
24) A rotating ramp with a slippery surface allows a maximum speed of 13 m/s for cars. Cars exit the ramp onto an exit ramp with the same angle of inclination as the first ramp. If we want the second exit ramp to allow a maximum speed of 26 m/s, the radius of the first ramp must be changed to:

A) One quarter
B) One half
C) Two times
D) Four times



Question Number	Correct choice	Grade
.1	A	2
.2	D	3
.3	C	5
.4	C	5
.5	C	5
.6	C	6
.7	D	6
.8	A	5
.9	B	3
.10	D	5
.11	C	5
.12	A	6
.13	A	5
.14	B	5
.15	C	5
.16	A	2
.17	C	2
.18	C	4
.19	A	3
.20	B	5
.21	D	3
.22	B	5
.23	D	2
.24	D	3
Total		100

SOLUTIONS

Chapter 1:

1- $[ML^{-1}T^{-2}]$. **2**- $T \propto L^{\frac{1}{2}} \times g^{-\frac{1}{2}}, T \propto \sqrt{\frac{L}{g}}$. **3**- B. 4- B.

Chapter 2:

1a- 12.6 km/h at 71.6° north of east. **1b**- minutes. **2a**- 19.5° west of north. **2b**- 13.3 minutes. **3a**- 261 km/h. **3b**- 15.3° south of east. **4**- 866 km/h. **5**- 2.92 m/s, 7.9° above the east direction.

Chapter 3:

Exercises: **1**- $\boldsymbol{a} = (29.3\hat{\imath} - 1.88\hat{\jmath})m/s^2$. **2a**- $a = \frac{F}{m_A+m_B+m_C}$ **2b**- $a = \frac{F-(m_A+m_B+m_C)gsin\theta}{m_A+m_B+m_C}$. **3a**- $a = \frac{mg}{m+M}$. **3b**- $T = \frac{Mmg}{m+M}$. **4a**-$a = \frac{(m_1-m_2\sin\theta)g}{m_1+m_2}$. **4b**- $T = \frac{m_1m_2g(1+\sin\theta)}{m_1+m_2}$. **5**- $a = 1.82$m/s²,5b- $T = 21.8$N. **6**- $a = 1.12$m/s². **7**- $\mu_s = \tan\theta_c$.

Additional problems: **1**- Weight = 637 N; Scale reads 510 N or 52 kg. **2**- 400N. **3a**- m/s² down the slope **3b**- $v = 16.2$m/s **4a**- Weight = 196 N (down); Normal force = 196 N (up) **4b**- 294 N, 98N. **5**- 3.63 × 10³ N. **6**- 2.45 m/s² downward. **7**- 1.41 × 10³ N. **8**- 17.0° (backward). **9**- 2.6m. **10**- 2.7×10^3N (upward) **11a**- $a_1 = 1.68\ m/s^2, a_2 = 0.840\ m/s^2$ **11b**- 13.4N. **12a**- 350N **12b**- 1.96 m/s² **13**- 20N,10N. **14a**- 180N , **14b**- 644N

Chapter 4:

Exercises: **1**- 219m/s, 219 m/s². **2**- 14.2 N. **3a**- 3.50m/s, **3b**- 19.6N. **4a**- 6.05 m/s^2, **4b**- 5.62 m/s^2, **4c**- 8.26 m/s^2, $\phi \approx 43°$

Additional problems: **1**- 1684 m/s, 1 hour 48 minutes. **2**- $a_c = 83.8\ m/s^2$ **3a**- $F_c = 3920N, f_{smax} = 5880N\ so\ no\ skid$. **3b**- $F_c = 3920N, f_{smax} = 2450N\ so\ the\ car\ will\ skid$. **4**- $v = \sqrt{gLsin\theta tan\theta}$. **5a**- $tan\theta = \frac{v^2}{gr}$, **5b**- 22°. **6a**-$3.80mg$, **6b**– $1.80mg$. **7a**- $0.74m/s^2$, **7b**- $0.62m/s^2$, **7c**- $0.96m/s^2$.