National Science and Mathematics Olympiad

First Stage- Physics

CONTENTS

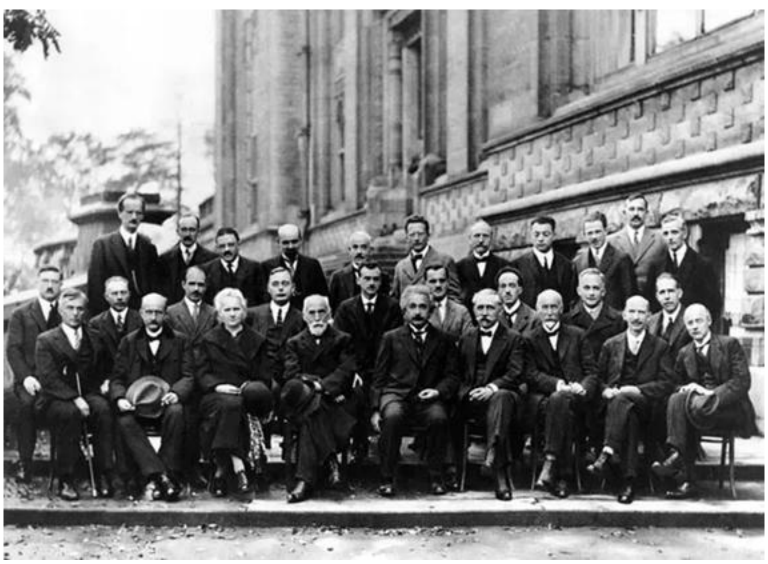
1 Physics and Its Nature

The ancient human tried early to understand and control the natural phenomena around him, such as the motion of objects and the ignition of fire, and he devised tools that help him move heavy objects and ways to ignite fire and benefit from it, and he made tools that help protect him from predators and in catching prey and in accomplishing his work and facilitate his life.

Simply put, what he did is physics in its broadest sense. In this chapter, you will learn about physics and its nature.

1.1 What is physics?

What comes to your mind when you hear the word "physics"? Perhaps you imagine a blackboard with many physical mathematical equations written on it, or you may remember pictures of famous physicists that you have heard a lot about, such as Isaac Newton or Albert Einstein, and you may think about the many technical applications developed by physics, such as the laptop computer and modern communication devices, artificial satellites and many others.



The smartest picture in history, taken at the Solvay Conference in 1927, gathered a large number of the most famous physicists, such as Einstein, Marie Curie, Dirac, Heisenberg, Pauli, Schrödinger, Bohr, Compton, and others.

In fact, what you imagined is correct to some extent and shows important aspects of physics, but physics in its broadest sense is: a branch of science concerned with the study of the natural world, matter and energy, and how they are related.

Think 1.1:

What do we mean by matter, and what do we mean by energy? Give examples of them. Could you list some differences between them?

The word physics is derived from a Greek word " φύσις " (Physica), which means "nature". So, physics is one of the most important aspects of our lives. Whatever we do, there is physics. We apply the principles of physics in our everyday life activities.

1.2 Physics And Natural Phenomena

The connection of matter with energy appears in the natural phenomena around us, such as the motion of objects, lightning, thunderbolts, magnetic attraction of things, water waves, and many others. Physics is mainly aimed at:

1. Understanding and explaining these phenomena through the development of laws and theories. An example of this is what the scientist Isaac Newton did in developing equations and laws of motion, which helped us a lot in the mathematical calculation of the speed of an object after a certain period of time had passed, and in calculating the net force acting on it.
2. Utilising understanding of natural phenomena in making modern applications. Such as making cars, planes, spacecraft, lightning rods, and others.
3. Predicting natural phenomena and their future results. Example: predicting the times of eclipses, eclipses, earthquakes, the structure of the universe, and others.

Exercise 1.1: We mentioned earlier that physics studies the relationship between matter and energy. In his famous experiment, Newton studied the decomposition of the white light spectrum using a glass prism. What matter did Newton study, what energy, and how was the connection between them?

1.3 Physics And Other Sciences

For more than two thousand years, physics, chemistry, biology, and certain branches of mathematics were part of natural philosophy, but during the scientific revolution of the seventeenth century, these sciences became separated but closely linked. Physics is the basic science because it studies nature in general and provides the basis for all other sciences. It also manufactures measuring devices and invents technical applications that benefit these sciences. Be proud of your study of physics.

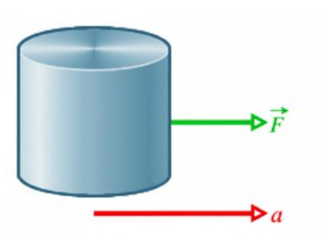
Quantum technology has become at the forefront of the most important and exciting fields that bring together sciences such as physics, biology, chemistry, engineering, and many other fields. This technology gave great hope for scientific revolutions in the near future, as it will change the direction of technology in many applications.

1.4 PHYSICS AND MATH

Physics is closely related to mathematics, and the relationship between them can be briefly described as follows: Mathematics is the language of physics in accurately expressing its description and interpretation of natural phenomena.

This Correlation appears in several forms, the most important of which are:

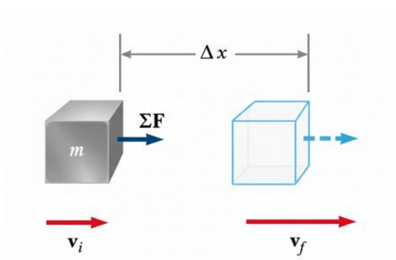
1. Physics uses mathematical equations to clarify the relationship between the physical quantities, or to calculate unknown quantities, for example:



A. $\sum \boldsymbol{F} = m\boldsymbol{a}$ Newton's second law of motion

It states: the acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass.

Note: acceleration means how much the velocity change in each second.

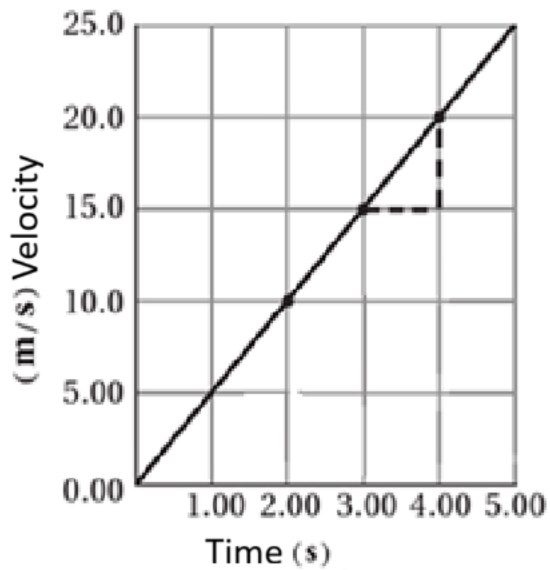


B. $\boldsymbol{v_f} = \boldsymbol{v_i} + \boldsymbol{a_x}t$ Equation of motion in a straight line

It is an important equation that helps us to find the instantaneous velocity of an object $\boldsymbol{v_f}$ at a specific time t in terms of its initial velocity $\boldsymbol{v_i}$ and its acceleration $\boldsymbol{a_x}$.

2. Physics uses mathematical graphs a lot, in order to accurately describe some phenomena and situations, for example the corresponding graph determines how the velocity v of an

object moving in a straight line changes with time t, and through the graph you can calculate the velocity of the object at each time moment shown in graph, and also conclude that the relationship between them is proportional (they increase together regularly).



1.5 Basic and Derivative Physics Quantities

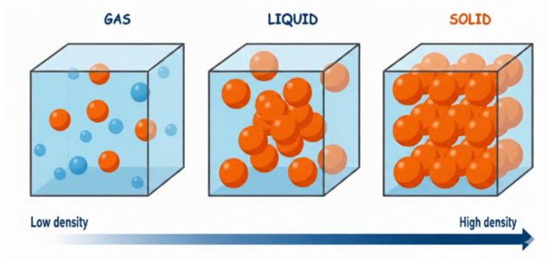
Physics studies the properties of matter, which we call "Physical Quantities".

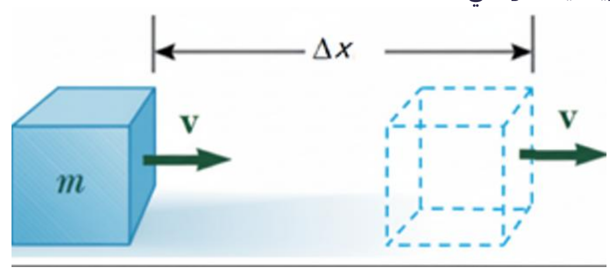
Physical quantities are of two types:

Derivative Quantities	Basic Quantities
They are the quantities defined by other basic quantities.	They are the quantities defined by themselves and are the basis for the derivation of other quantities.
Ex: velocity, density.	Ex: length (distance), mass, temperature

Exercise 1.2: Try to define velocity and density in terms of other basic quantities

Density	Velocity





1.6 Units In Physics

Suppose your friend asked you about the distance between your home and your grandfather's home, and you answered that it is 200. Is this answer sufficient to understand the distance between them accurately? Of course not, the distance could be 200 meters ($200\ m$), 200 kilometres ($200\ km$), or 200 miles ($200\ miles$). We call meters, kilometres, and miles units of measure, and they are necessary, as you noted, to accurately determine the distance. In fact, this is a fundamental difference between mathematics and physics. Mathematics deals with abstract numbers, while physics is concerned with writing the unit of the numerical value of any measurement.

It was agreed to establish an international system of measurement called "the International System of Units" with the aim of unifying units of measurement worldwide, and it was referred to by the symbol (SI). This system identified seven basic quantities in physics with the definition of their units of measurement, and these quantities are:

Basic Quantity		Basic Unit	
Name	Symbol	Name	Symbol
Length	ℓ	Meter	**m**
Mass	m	Kilogram	**kg**
Time	t	Second	**s**
Electric Current	I	Ampere	**A**

Temperature	T	Kelvin	**K**
Amount of Substance	n	Mole	**mol**
Luminous intensity	E	Candela	**Cd**

Note that symbols of physical quantities are written in italicised letters, and symbols of physical units are written in non-italicised letters.

Fun Fact

In A.D. 1120, the king of England decreed that the standard of length in his country would be named the yard and would be precisely equal to the distance from the tip of his nose to the end of his outstretched arm. Similarly, the original standard for the foot adopted by the French was the length of the royal foot of King Louis XIV. This standard prevailed until 1799, when the legal standard of length in France became the meter.

Non-fundamental physical quantities are derived quantities, and their units are composed of two or more basic physical units, such as velocity (ms^{-1}), acceleration (ms^{-2}), density ($\mathrm{kg} \cdot \mathrm{m}^{-3}$) and many others.

Some units of derived quantities are relatively long, and for their abbreviation they are named after the names of the scientists who contributed to their development, then the first letter of the scientist's name in the English language was taken in the capital letter to express that unit, for example, the unit of force measurement was named "Newton" relative to the scientist Newton, and it was abbreviated like this (N).

Exercise 1.3: An electric heater is used to boil water. When the switch is turned on, the electric current in the heating element produces heat energy. The temperature of the water increases steadily until it starts to boil after 15 minutes. If another heater with a greater power is used, the time taken to boil the same volume of water would be less than 15 minutes. From the above description, identify the physical quantities. Then, classify these quantities into base quantities and derived quantities.

1.6.1 Conversion of Units

The basic idea of the conversion is to multiply by the conversion factor, which is a fraction whose value is one, and is written to allow units to be shortened. Example: conversion factors between kilograms and grams, minutes and seconds:

$$1 = \frac{1000\text{g}}{1\text{kg}}\ , 1 = \frac{1\text{kg}}{1000\text{g}}$$
$$1 = \frac{1\ \text{min}}{60\ \text{s}}\ , 1 = \frac{60\ \text{s}}{1\ \text{min}}$$

Exercise 1.4: Convert the following:

500g to kg.

24 minutes to seconds.

$30\ \text{ms}^{-1}$ to km h^{-1}.

1.6.2 Prefixes

In physics, we sometimes need to write some values of quantities using prefixes, especially very large or very small values, in order to make it easier to write and understand them more clearly.

For example, $30000\ \text{m}$ is in meters, we can write it in kilometres as follows: $30\ \text{km}$.

Also, $0.000001\ \text{s}$ in the second unit, we can write it in the unit of microseconds as follows: $1\ \mu\text{s}$

The table shows some of the prefixes used in physics:

Small Prefixes		
Value	Symbol	Name
10^{-2}	c	Centi
10^{-3}	m	Milli
10^{-6}	μ	Micro
10^{-9}	n	Nano

large Prefixes		
Value	Symbol	Name

10^3	k	kilo
10^6	M	Mega
10^9	G	Giga

To write values with or without prefixes, we use the conversion factor:

1. To add the prefix: $\dfrac{\text{prefix that we want to write}}{\text{prefix value}}$

2. To remove the prefix: $\dfrac{\text{prefix value}}{\text{prefix that we want to omit}}$

Exercise 1.5:

Write 2 µs in the unit of seconds.

Write $6.7\times10^{-8} g$ in the unit of (ng).

Write 0.7ng in the unit of (kg).

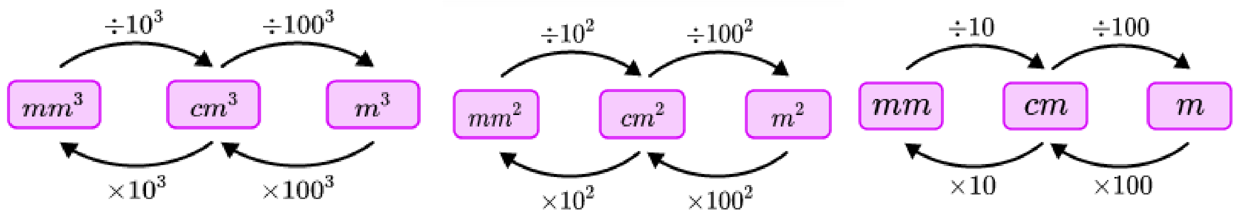
It is also useful to familiarise yourself with the rules for converting between units of length, area and volume:

Length units: $1\text{m}=10^2\text{cm}=10^3\text{mm}$

Area units: $1\text{m}^2 = 10^4\text{cm}^2 = 10^6\text{mm}^2$

Volume units: $1\text{m}^3 = 10^6\text{cm}^3 = 10^9\text{mm}^3$

The litre: $1\text{L}=10^3\text{cm}^3 = 10^{-3}\text{m}^3$



1.7 SIGNIFICANT DIGITS

Measurements resulting from the use of tools and devices are approximate, so they are written in the form of significant digits, and the last number on the right in the measurement result is uncertain. To clarify, "significant" digits are the numbers that are reliable in a measurement.

For example, to measure an object, a one-meter ruler gives a measurement that can be expressed as $24.5\mathrm{cm}$. However, from a more precise ruler, we can express the measurement as $24.45\mathrm{cm}$. The extra digit is not arbitrary but does have a meaning; this means that in physics $20m$ and $20.0m$ are not the same!

When multiplying or dividing multiple quantities, the result should have the same number of significant figures as the measurement with the lowest significant figures.

1.8 EXERCISES

1. A car travels at $90\mathrm{kmh}^{-1}$. Convert this speed to $\mathrm{m\ s}^{-1}$.
2. Write $5\mathrm{L}$ in units of mm^3
3. How many micrometres make up 1.0 km?
4. The density of water is $1\ \mathrm{g\ cm}^{-3}$. What is this density in $\mathrm{kg\ m}^{-3}$?
5. Write the value $0.0056\ \mathrm{m}$ using a suitable SI prefix.
6. Perform the calculation and round to the correct number of significant figures:
   $12.45\ \mathrm{m}\ \times\ 3.2\ \mathrm{m}$.
7. Perform the calculation and round to the correct number of significant figures:
   $105.4\ \mathrm{g}\ \div\ 25.2\ \mathrm{mL}$. Also, answer in $\mathrm{kg\ m}^{-3}$.
8. The area of a rectangle is found by multiplying its length by its width.
   If length $= 5.0\ \mathrm{cm}$ and width $=\ 3.00\ \mathrm{cm}$, what is the area with the correct number of significant figures?
9. If the side of a cube is $\mathbf{0.50\ m}$, what is the volume in both $\mathbf{m}^3$ and $\mathbf{L}$?

10. Earth is approximately a sphere of radius $6.37 \times 10^6 \mathrm{m}$. What are (a) its circumference in kilometres, (b) its surface area in square centimetres, and (c) its volume in litres?

2 Introduction to Fluids

The crown of King Hiero: the king of Syracuse had given a craftsman a certain amount of gold to be made into an exquisite crown. When the project was completed, a rumour surfaced that the craftsman had substituted a quantity of silver for an equivalent amount of gold, thereby devaluing the crown and defrauding the king. Archimedes was tasked with determining if the crown was pure gold or not. The Roman architect Vitruvius relates the story: While Archimedes was considering the matter, he happened to go to the baths. When he went down into the bathing pool, he observed that the amount of water which flowed outside the pool was equal to the amount of his body that was immersed. Since this fact indicated the method of explaining the case, he did not linger, but moved with delight, he leapt out of the pool, and going home naked, cried aloud that he had found exactly what he was seeking. For as he ran, he shouted in Greek: Eureka! Eureka!

Archimedes came up with one of the most important fluid principles, which was named "Archimedes' principle". In this chapter, you will learn about other concepts, principles and laws in fluids.

2.1 What are Fluids?

The word fluids refers to every substance that has the property of flowing or diffusion, and thus includes: liquids and gases. The origin of the word fluid is from fluidity, which is physically: not maintaining a constant shape. The basic properties common to liquids and gases are:

1. It has no constant shape.
2. Its particles are far apart.
3. Its molecules move freely:
    a. In liquids: molecules move translationally within the volume of the liquid.
    b. In gases: molecules move freely, in any size available.

Exercise 2.1: Can you name important differences between the properties of liquids and gases?

2.2 Mass, Volume, and Density

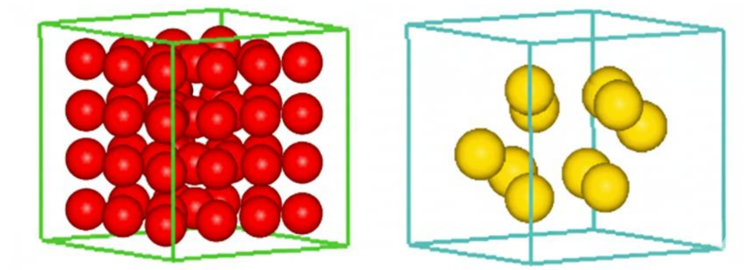
Mass, volume, and density are distinctive properties of any sample of matter, and it is very important to be aware of their meaning.

Density	Volume	Mass

Mass of a unit volume of a sample It is measured in kg/m^3	The portion of space that a sample of matter occupies. It is measured in m^3	The amount of matter in a sample of it. It is measured in kg

If the particles of a substance are closer to each other, then it is denser.

Exercise 2.2: Which of the two samples is denser?



2.2.1 Definition of Density:

Density is defined as the mass per unit of volume, which can be expressed by the equation

$$\rho = \frac{m}{V}$$

where:

m is the mass(Kg)

V is the volume (m^3)

ρ is the density $(\mathrm{kg/m}^3)$

Density is a characteristic of any pure substance; that is, each type of substance has its own density, such as iron, copper, gold, etc.

The table shows the densities of some substances at standard situations:

Substance	Density (kg/m³)	المادة
Air	1.29	الهواء
Ice	0.917×10^3	الثلج
Aluminum	2.70×10^3	الألومنيوم
Iron	7.86×10^3	الحديد
Benzene	0.879×10^3	البنزين
Lead	11.3×10^3	الرصاص
Copper	8.92×10^3	النحاس
Mercury	13.6×10^3	الزئبق
Oak	0.710×10^3	خشب البلوط
Fresh water	1.00×10^3	الماء العذب
Oxygen gas	1.43	غاز الأكسجين
Glycerin	1.26×10^3	الجلسرين
Pine	0.373×10^3	خشب الصنوبر
Gold	19.3×10^3	الذهب
Platinum	21.4×10^3	البلاتين
Helium gas	1.79×10^{-1}	غاز الهيليوم
Seawater	1.03×10^3	ماء البحر
Hydrogen gas	8.99×10^{-2}	غاز الهيدروجين
Silver	10.5×10^3	الفضة

Exercise 2.3: What is the volume of helium (density $0.179\text{kg}\cdot\text{m}^{-3}$). that has the same mass as $5.0\text{m}^3$of nitrogen (density $1.25\text{kg}\cdot\text{m}^{-3}$).

Exercise 2.4 Arrange these iron solid objects ascending a) according to their densities b) according to their masses.

a. A ball of radius r

b. A cube of side length r

c. A cylinder of height r and radius r

d. A cuboid of length r, width $2r$, and height $0.5r$

2.3 Pressure

The plane shown in the figure is standing on a runway of an airport, and the plane weight within 420 tonnes. Intuitively, this weight is distributed over the runway area, and we call the part of its weight affecting on each unit area (1m^2) of the runway: pressure.

2.3.1 Definition of Pressure

When a force acts on a surface, we can say that the force is exerting pressure on it. In physics, pressure is defined as the perpendicular force on the surface per unit area, and it can be calculated by the equation:

$$P = \frac{F}{A} \qquad (1)$$

Where:

F is the force (N)

A is the area (m^2)

P is the pressure $(\mathrm{Nm}^{-2}$=Pa) (Pascal)

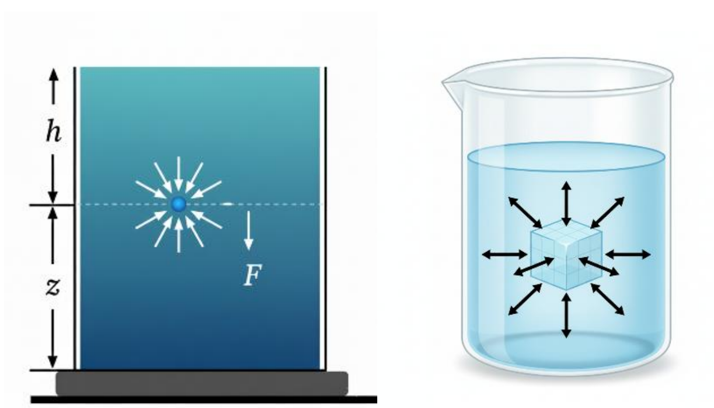
Exercise 2.5: Why do skaters use special shoes instead of regular shoes?

Exercise 2.6: Explain: In the figure, the guy is not harmed despite being stretched out on a layer of nails.

2.3.2 Pressure in Fluids

Have you tried scuba diving? The diver feels that water exerts pressure on all parts of his body, and his feeling of pressure increases if he dives deep. This can also be witnessed when climbing a mountain, where the higher you go, the lower the pressure you feel.

In fact, all fluids (liquids and gases) exert pressure on objects immersed in them. The pressure of the fluid affects in all directions on an object that immersed in it.



Experimentally, it was found that the pressure of a fluid at a point increases with the increase of:

a. The depth of the point below the surface of the fluid h.

b. the density of the fluid ρ_f.

Therefore, the fluid pressure at a point is calculated by the equation:

$$P_f = \rho_f hg \qquad (2)$$

Where:

P_f is the pressure of fluid (Pa)

ρ_f is the density of the fluid (kg·m^{-3})

h is the depth of the point below the surface (m)

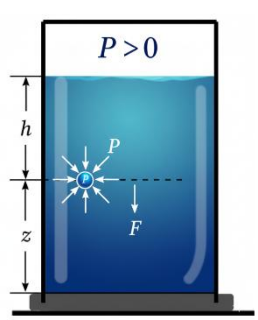
2.4 THE EXPOSURE OF THE ATMOSPHERE

When the fluid is exposed to the atmosphere, then the atmosphere will exert pressure on it, this is true for all fluids too, where fluids can exert pressure on each other.

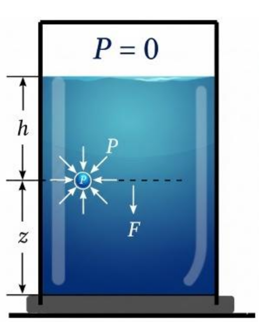
Exercise 2.7:

Calculate the total (absolute) pressure at the point P:

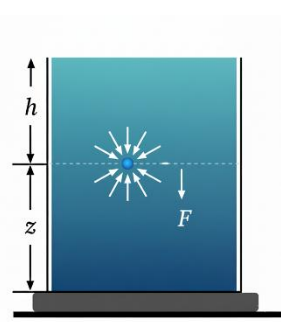
Fluid is not exposed to the atmosphere and pressure above it: $P > 0$



Fluid is not exposed to the atmosphere and pressure above it: $P = 0$



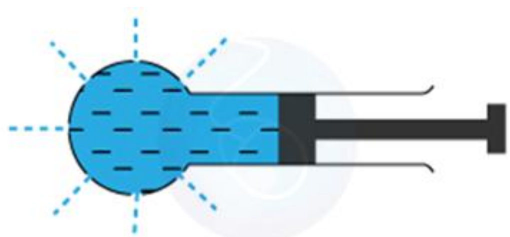
Fluid exposed to the atmosphere



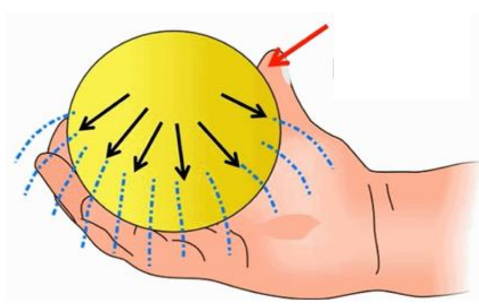
2.5 Pascal's Principle

Pascal's law (also Pascal's principle or the principle of transmission of fluid-pressure) is a principle in fluid mechanics given by Blaise Pascal that states that a pressure change at any point in a confined incompressible fluid is transmitted throughout the fluid such that the same change occurs everywhere. The law was established by French mathematician Blaise Pascal in 1653 and published in 1663.

For example, when the plunger is pushed in, the water squirts equally from all the holes. This shows that the pressure applied to the plunger has been transmitted uniformly throughout the water.



Also, when you press your finger on a balloon filled with air, the pressure on the surface of the balloon increases by the same value in all directions.



2.5.1 Hydraulic Press

The hydraulic press operates according to Pascal's principle. We see it in our daily life, such as a car lift used in a service station. When a force F_1 is applied to the small piston, the piston is affected by pressure: $P = F_1/A_1$. This pressure is transmitted through the fluid in the lever and exerts the same pressure on the large piston: $P = F_2/A_2$. So:

$$F_1/A_1 = F_2/A_2 \quad (3)$$

Then:

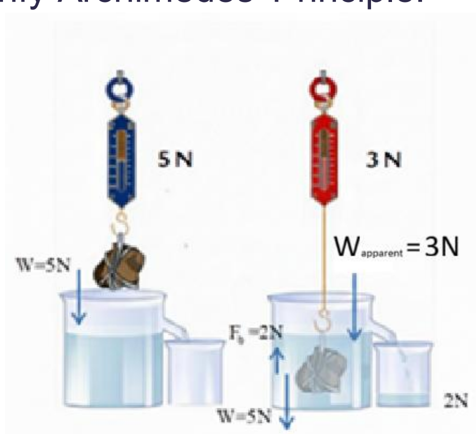
$$F_2 = F_1 \frac{A_2}{A_1} \quad (4)$$

So, A large force is created that affects the large piston, and it is capable of lifting heavy objects such as cars. The quantity (A_2/A_1) is called: mechanical advantage or force multiplication coefficient.

2.6 Archimedes' Principle

From our daily observations, objects appear to be lighter under water. For example, it is difficult for us to lift a heavy rock from the surface of the earth, while we can do so easily if it is submerged under water. This indicates that it is affected by an upward force that reduces its weight, called the Buoyant force and symbolised by F_B. The discovery of the principle is attributed to Archimedes (212-287) BC.

2.6.1 Experiment to verify Archimedes' Principle:



The basin is filled to the edge with water. We weigh the object in the air (reading of the scale): $W = 5N$

When the object is immersed in water, Water flows into the trough. The weight of the object inside the water (reading of the scale): $W_{apparent} = 3N$. This means that the buoyant force is:

$$F_B = W - W_{apparent} \quad (5)$$

We can see that the weight of the displaced water is equal to the buoyant force. The volume of the displaced water is equal to the volume of the whole object. Therefore, by the displaced fluid we always mean: the weight of a volume of the fluid equal to the volume of the immersed part of the object.

Statement: Any object, totally or partially immersed in a fluid or liquid, is buoyed up by a force equal to the weight of the fluid displaced by the object.

$$F_B = \rho_{fluid} \times V_{displaced} \times g \tag{6}$$

Where:

F_B is the buoyant force (N).

ρ_{fluid} is the density of the fluid ($kg\ m^{-3}$).

$V_{displaced}$ is the volume of fluid displaced by the object (m^3).

g is acceleration due to gravity (ms^{-2}).

2.7 Exercises

1. A block of wood has a mass of 120 g and a volume of $150\ \mathrm{cm}^3$. Find its density in $\mathrm{g \cdot cm^{-3}}$ and $\mathrm{kg \cdot m^{-3}}$. Would it float in water?
2. A force of $300\ \mathrm{N}$ acts on an area of $0.15\ \mathrm{m}^2$. Find its pressure.
3. The pressure at a certain point in water is $3.0 \times 10^5\,\mathrm{Pa}$. If the water density is $10^3\mathrm{kg \cdot m^{-3}}$, find the depth of that point below the surface.
4. A $2\ \mathrm{m}^3$ block of wood is completely submerged in water (density = $10^3\mathrm{kg \cdot m^{-3}}$). Calculate the buoyant force acting on it.
5. An object weighs $40\ \mathrm{N}$ in air and $35\ \mathrm{N}$ when completely immersed in water. Find the buoyant force and the volume of the displaced water. (Water density = $10^3\mathrm{kg \cdot m^{-3}}$)
6. A hydrometer sinks deeper in oil than in water. What does this tell you about the densities of the two liquids?

7. A piece of wood immersed one-third of its volume if placed in water (the density of water is $10^3 \mathrm{kg \cdot m^{-3}}$) , While immersed half of its volume when put in oil, find the density of oil?
8. A cube has a density of $8.0 \times 10^2 \mathrm{kg \cdot m^{-3}}$and a height of $6\ \mathrm{cm}$. Part of it floats in a liquid whose density is $1.2 \times 10^3 \mathrm{kg \cdot m^{-3}}$. Calculate the length of the submerged part of the cube in the liquid.
9. An object with mass $170\ \mathrm{kg}$ displaces 85 liters of fresh water. find the buoyant force acting on it.
10. A solid cylinder with a density of $2.0 \times 10^3 \mathrm{kg \cdot m^{-3}}$ and a weight of $400\ \mathrm{N}$ in air, we immersed it in oil, so its weight became $300\ \mathrm{N}$. find the density of the oil.
11. A solid metal cylinder weighs $400\ \mathrm{N}$ in air, after immersing it in fresh water, its weight is $300\ N$. Calculate the volume of the cylinder. (Water density = $10^3 \mathrm{kg \cdot m^{-3}}$)
12. The total (absolute) pressure at the bottom of a swimming pool is $151{,}325\ \mathrm{Pa}$. If the atmospheric pressure is $101{,}325\ \mathrm{Pa}$, find the depth of the swimming pool.
13. An iceberg of density $9.2 \times 10^2 \mathrm{kg \cdot m^{-3}}$floats in seawater of density $1.03 \times 10^3 \mathrm{kg \cdot m^{-3}}$. If the total volume of the iceberg is $2000\ \mathrm{m^3}$, find the volume of ice above the water surface.

3 Motion in one dimension

To describe an event in nature, we express it using physical quantities, such as temperature, pressure, speed, and force. The effect of some of these quantities cannot be determined without knowing their direction, while others cannot be described by a direction. This distinction allows us to divide physical quantities into two types: scalar quantities and vector quantities.

3.1 Scalar and Vector Quantities

Scalar quantities are physical quantities that are fully described by their magnitude (value) only, and no direction can be assigned to them. Examples include temperature and time. It makes no sense to say "the temperature is 30^o upward" or "I waited for you for two hours toward the north."

Vector quantities are quantities that require both a magnitude and a direction to be fully described, such as force and displacement. Suppose a force of 5N acts on a stationary object. Will the object move up or down? Or will it move right or left? And if another force of 5N also acts, will the object move with greater acceleration, or will the two forces balance each other so the object doesn't move? Without knowing the direction, we cannot describe the effect of these physical quantities.

Important note: Vector quantities are written with an arrow or in bold to differentiate between them and the scalar quantities, ($\vec{A}$, $\boldsymbol{A}$). The magnitude of the vector is written with the normal font $A = |\vec{A}|$.

3.2 Vector Quantities in Three Dimensions

We live in a three-dimensional world, which means vector quantities exist in three dimensions. Fortunately, we can describe any vector quantity as the sum of three vector quantities, each lying along one of the three dimensions (axes). We can then study each vector independently of the others. At this stage, the requirement will be to study vector quantities in one dimension, and we will leave the analysis of vectors in multiple dimensions for later stages.

3.3 MOTION

The movement of objects – such as a football, a car, and even the sun and moon – is a clear part of our daily lives. The modern concept of motion became clear in the 16th and 17th centuries AD, with significant contributions from Galileo Galilei (1564-1642) and Isaac Newton (1642-1727), who helped formulate the modern understanding of motion.

The study of the motion of objects and the concepts related to force and energy includes a field called Mechanics, which is usually divided into two branches:

Kinematics	Dynamics
Describes the motion of objects in terms of position and time without considering their causes.	Describes the motion of objects by the effect of their causes, such as forces.

We will now discuss the translational motion of objects without rotation, specifically in one dimension (on a straight line).

3.3.1 Position

To describe the motion of an object well, physicists need to know several physical quantities about it, such as its position, displacement, velocity, and acceleration, which we will study successively, and we will start by knowing what we mean by position.

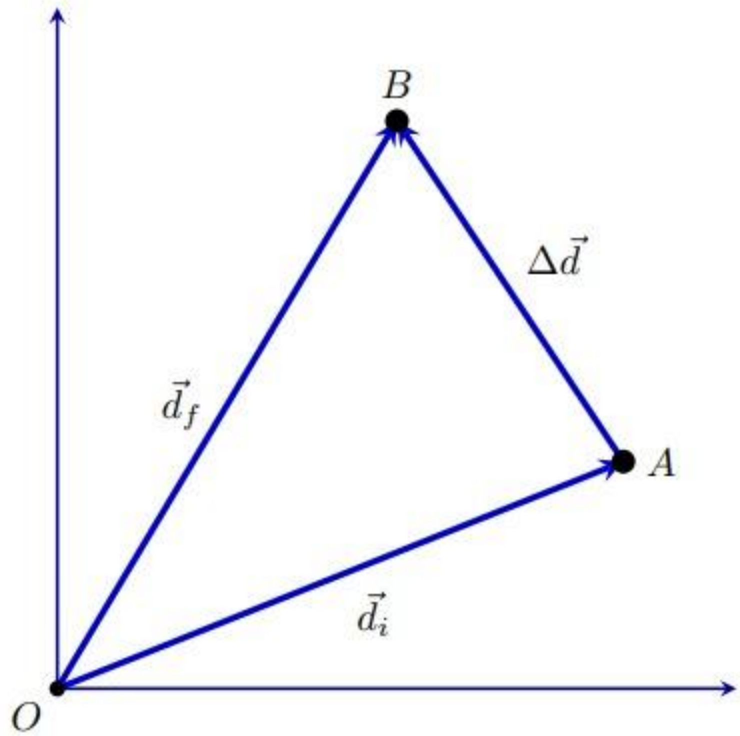
Position is defined as the location of an object at a particular moment relative to a reference point, often called the origin.

3.3.2 Distance and Displacement

	Distance	Displacement
Concept	The actual length of the path of the object's motion; it is a scalar quantity.	The change in the position of the body in a specific direction; it is a vector quantity.

Displacement is a vector connecting the endpoints of the path of an object. The displacement does not depend on the path taken, and it can be calculated by the equation:

$$\Delta\vec{d} = \overrightarrow{d_f} - \overrightarrow{d_\imath} \qquad (7)$$



Where:

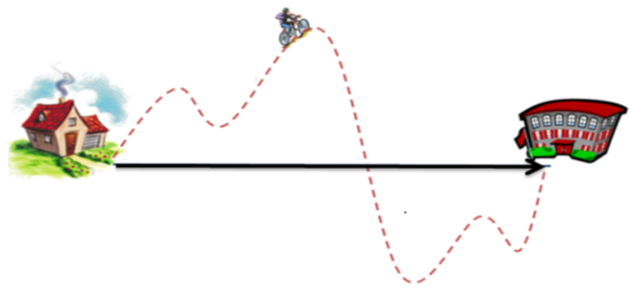
$\Delta\vec{d}$ is the displacement (m).

$\overrightarrow{d_f}$ is the final position vector (m).

$\overrightarrow{d_\imath}$ is the initial position vector (m).

Note that in the graph, the path wasn't taken into consideration, only the end points of the path (the start and the end points). While the distance "s" can be calculated only by calculating the length of the path taken by the object.

Exercise 3.1: A cyclist moves from his home to his school as shown in the figure. How do we distinguish between distance and displacement?



Exercise 3.2: A runner ran around a circle with a radius $r = 10.0\ m$. a) After he finished the first round, what was his displacement and distance? b) What will his displacement and distance be if he ran 10 rounds? c) What was his displacement and distance at a quarter round? d) And at half round? e) And at 3-quarters?

3.3.3 Speed and velocity

Velocity and speed measure how fast the object is moving, with the speed being a scalar quantity and velocity as a vector. So, we can define the following:

Average speed: is the rate of change of distance with respect to time. And can be calculated by the equation

$$\bar{v} = \frac{s}{t} \tag{8}$$

Where:

$\bar{v}$ is the average speed (ms^{-1}).

s is the total distance (m).

Δt is the time interval (s).

Average velocity: is the rate of change of position with respect to time. And can be calculated by the equation:

$$\vec{\bar{v}} = \frac{\Delta \vec{d}}{\Delta t} = \frac{\overrightarrow{d_f} - \overrightarrow{d_\iota}}{\Delta t} \tag{9}$$

Where:

$\vec{\bar{v}}$ is the average velocity (ms^{-1}).

d_i is the initial position (m).

d_f is the final position (m).

$\Delta\vec{d}$ is the displacement (m).

Δt is the time interval (s).

Exercise 3.3: If the runner in the last example completed the round in 12.5 s, what is his average speed and his average velocity?

3.3.4 Acceleration

Acceleration is the rate of change of velocity. It is a vector quantity, and its direction gives us insight into the object's motion. If it is in the same direction as the velocity, then the object is accelerating (speeding up), and if it is in the opposite direction, then the object is decelerating (slowing down).

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t} \tag{10}$$

Where:

$\vec{a}$ is the average acceleration (ms^{-2}).

$\vec{v}_i$ is the initial velocity (ms^{-1}).

$\vec{v}_f$ is the final velocity (ms^{-1}).

$\Delta\vec{v}$ is the change of velocity (ms^{-1}).

Δt is the time interval (s).

Exercise 3.4: A rabbit runs along the x-axis. What is the sign of its acceleration if it is moving (a) in the positive direction with increasing speed, (b) in the positive direction with decreasing speed, (c) in the negative direction with increasing speed, and (d) in the negative direction with decreasing speed?

3.4 Graphics Calculations

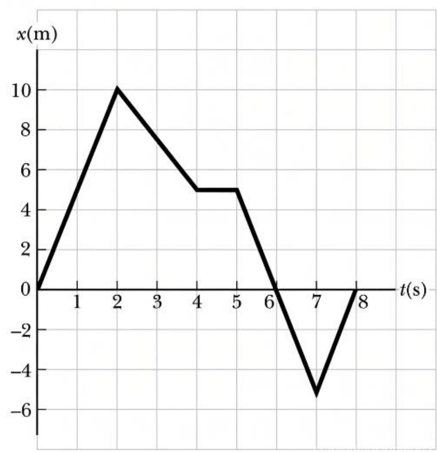
Algebraic equations are essential tools for solving problems in kinematics, yet graphical analysis offers a more intuitive and visually rich understanding of motion. By interpreting the slopes and areas of graphs, we can derive fundamental quantities such as displacement, velocity, and acceleration, and see how they relate to one another.

3.4.1 The position-time graph

This graph shows how the position of the object changes with time. From this graph, it is easy to extract the distance and displacement based on their definitions. Where the displacement only requires the endpoints, and the distance is calculated by summing all the distances travelled by the object.

Velocity is the rate of change of the position, which is the slope of the position-time graph.

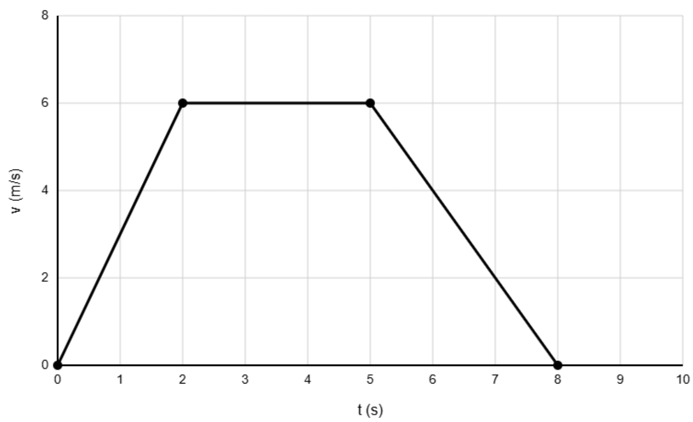
Exercise 3.5: In the following graph, a) determine the position at $t = 7.0\ s$ b) determine the average velocity in the interval between $t = 2.0\ s$ and $t = 4.0\ s$. c) determine the average velocity in the interval between $t = 2.0\ s$ and $t = 6.0\ s$.



3.4.2 The speed-time graph

This graph shows how the speed changes with time. In this graph, the distance can be obtained by calculating the area under the graph (the area is speed times time, which is the equation to calculate distance).

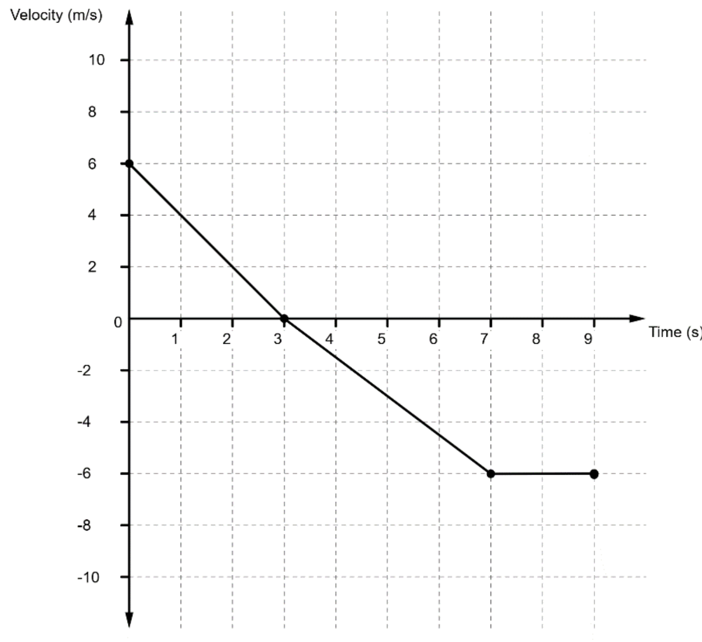
Exercise 3.6: The following graph shows the speed of an object versus time. From this graph, a) determine the speed at $t = 4.0\ s$. b) Determine the distance travelled from $t = 0.0\ s$ to $t = 8.0\ s$. c) Determine the distance travelled when the speed was constant.



3.4.3 The velocity-time graph

Similar to the speed-time graph, we can see the velocity and how it changes with time. This time, the area under the graph is the displacement. In addition, the slope of the graph is the change of velocity divided by the time interval, which is exactly the acceleration.

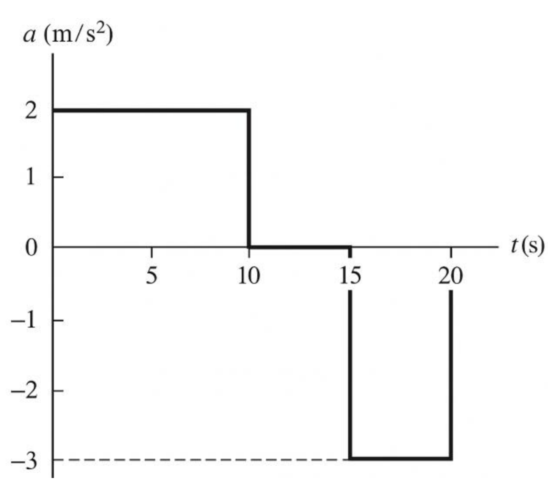
Exercise 3.7: The following graph shows the velocity of an object versus time. From this graph, a) Determine the velocity at $t = 1.0\ s$. b) At what second did the object change its direction? c) Determine the value and the direction of the average acceleration for the interval from $t = 0.0\ s$ to $t = 10.0\ s$. d) Determine the total displacement of the object for the interval from $t = 0.0s$ to $t = 7.0s$.



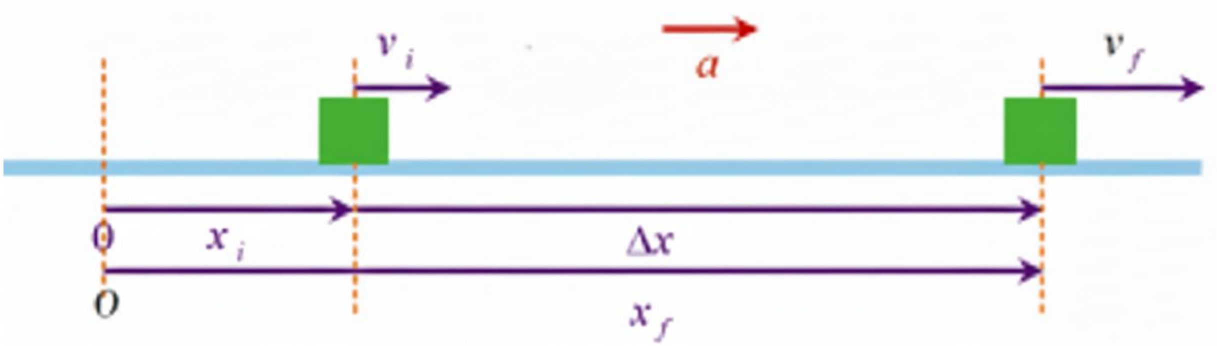
3.4.4 The acceleration-time graph

The acceleration-time graph shows how the acceleration changes with time, and the area under the curve gives the velocity difference.

Exercise 3.8: From the acceleration versus time graph, a) determine the direction and value of the acceleration at $t = 7.0\ s$. b) Calculate the change in the velocity from $t = 0.0\ s$ to $t = 20.0\ s$. c) When was the velocity constant?



3.5 Equations of Motion



How can we describe the motion of the object in the figure above, which moves in a straight line from the position on the left to the position on the right toward the axis with constant acceleration? We can describe the motion of this object using the following equations of motion.

$$v_f = v_i + at \tag{11}$$

$$\Delta x = v_i t + \frac{1}{2}at^2 \tag{12}$$

$$\Delta x = v_f t - \frac{1}{2}at^2 \tag{13}$$

$$v_f{}^2 = v_i{}^2 + 2a\Delta x \tag{14}$$

$$\Delta x = \left(\frac{v_f + v_i}{2}\right)t \tag{15}$$

Where:

O is the origin.

Δx is the displacement in the x-axis (it can be any axis) (m).

x_f is how far the object is from the origin initially (m).

x_i is how far the object is from the origin initially (m).

v_i is the initial velocity (ms^{-1}).

v_f is the final velocity (ms^{-1}).

a is the acceleration (ms^{-2})

t is the time interval (s).

3.5.1 Problem-solving skills using equations of motion

1) If the motion is in one direction: Consider it the positive direction of motion, whatever it is.

And the signs of displacement, velocity, and acceleration are positive in this direction and negative in the other direction.

2) If the motion is in more than one direction on one line: Consider one of the directions as positive (e.g., to the right or upward) and the other negative (to the left or downward).

And the signs of displacement, velocity, and acceleration are positive in the positive direction and negative in the other direction.

3) Use the appropriate equation in which all quantities are known except the quantity to be calculated. Note that you need to know 3 variables to determine the other 2.

4) If the problem contains several accelerations, apply the equations for each acceleration stage separately.

5) You need the number of equations to equal the number of unknowns.

Exercise 3.9: A truck covers $40.0\ m$ in $8.50\ s$ while smoothly slowing down to a final speed of $2.80 ms^{-1}$. (a) Find its original speed. (b) Find its acceleration.

3.6 Free Fall

Free fall is considered one of the examples of one-dimensional motion with a constant acceleration in nature, and it is defined as the motion of an object under the influence of Earth's gravity only, with the ignorance of air resistance, in a line perpendicular to the reference surface (the ground). We usually consider the upward direction as the positive one. The value of the acceleration is: $g = 9.8\ \mathrm{ms}^{-2}$ during ascent and descent.

The displacement and velocity of the body can be calculated at any moment during the period of ascent or descent, by utilizing the equations of motion and considering the acceleration in the equations of motion as $a = -g$:

$$v_f = v_i - gt \tag{16}$$

$$\Delta y = v_i t - \frac{1}{2} g t^2 \tag{17}$$

$$\Delta y = v_f t + \frac{1}{2} g t^2 \tag{18}$$

$$v_f{}^2 = v_i{}^2 - 2g\Delta y \tag{19}$$

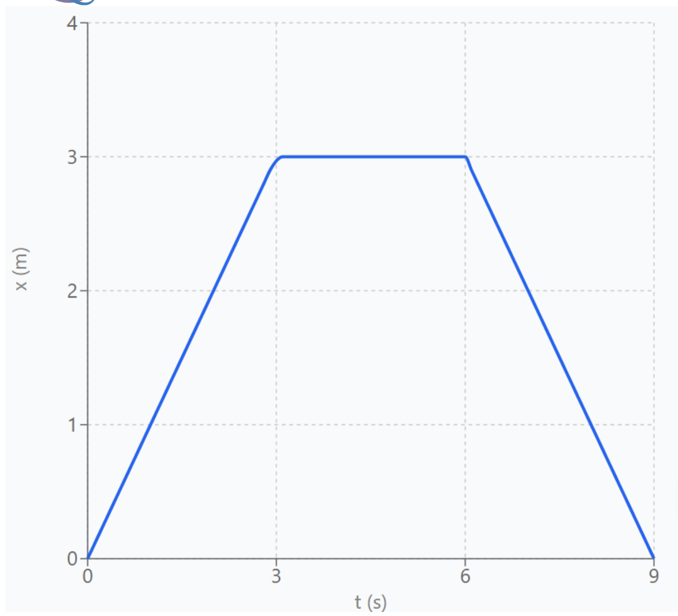
$$\Delta y = \left(\frac{v_f + v_i}{2}\right) t \tag{20}$$

It is also helpful to remember that the velocity at the highest point is $v = 0$. Otherwise, it will go even higher.

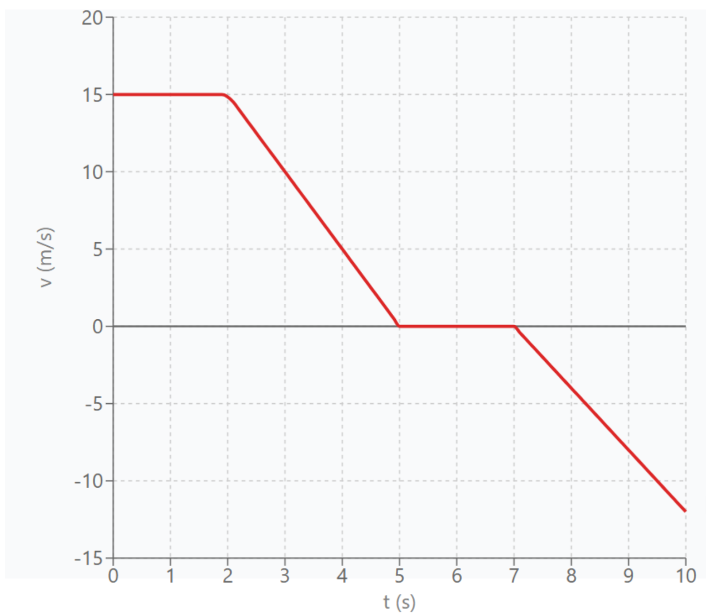
Exercise 3.10: A ball was thrown next to a building, and a man in the building saw the ball from his window. After 3 seconds, he saw it again. Was the ball's speed larger in the ascending or descending? And find the velocity in both cases.

3.7 Exercises

1. A person walks $4\ \mathrm{km}$ East, then $3\ \mathrm{km}$ North. Calculate the total distance travelled and the magnitude of his displacement.
2. A car travels from city A to city B with a constant speed of $100\ \mathrm{km/hr}$, and then back to city A with a constant speed $80\ \mathrm{km/hr}$. Calculate its average speed and average velocity for the entire trip.
3. A car accelerates from $20ms^{-1}$ to $30ms^{-1}$ in 5 seconds. What is its acceleration? And what was its displacement?
4. A cyclist slows down from $10ms^{-1}$ to a stop in 4 seconds. What is his acceleration? And what was his displacement?
5. An object's motion is described by the position-time graph below. Describe its motion in words (e.g., constant speed, at rest, etc.) for each segment.



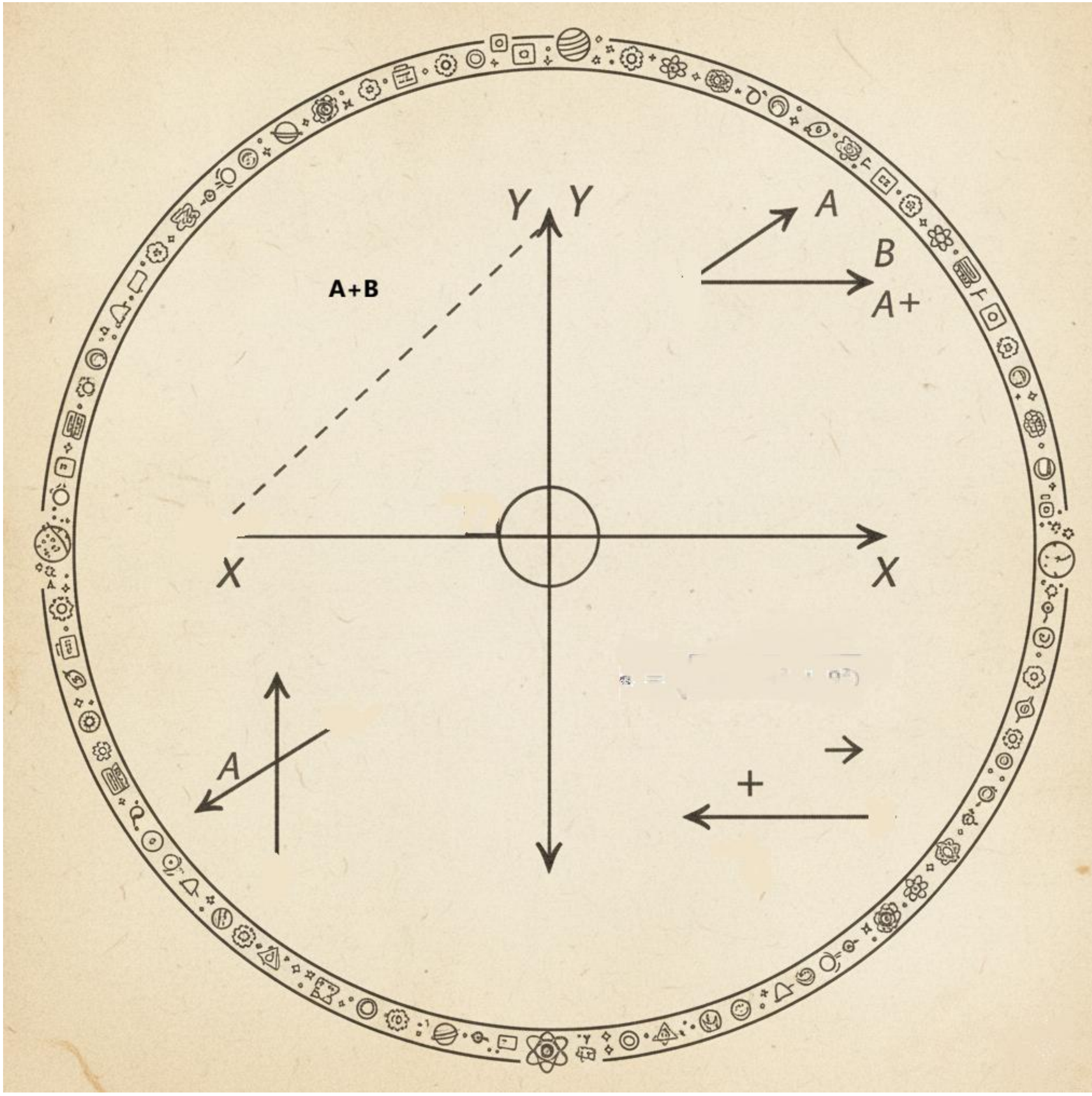
6. An object's motion is described by the velocity-time graph below. Find (a) the acceleration in each segment, and (b) the total displacement.



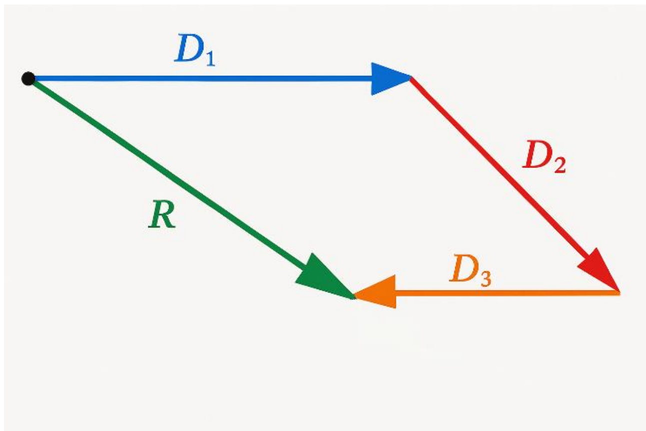
7. A ball is thrown vertically upward with an initial velocity of $20ms^{-1}$. How high does it go? How much time does it take to reach its maximum height?
8. A truck moving at $25ms^{-1}$ applies its brakes and comes to a stop in 10 seconds. What is its acceleration, and how far does it travel during this time?
9. A car accelerates uniformly from rest and covers a distance of $100\ m$ in 5 seconds. Find its acceleration and its final velocity.

10. A stone is dropped from a cliff. It hits the ground after 3 seconds. What is the height of the cliff?
11. A ball is thrown vertically upwards with a speed of $15ms^{-1}$ from the ground. Calculate the time it takes to return to the ground.
12. Two cars, A and B, are moving in the same direction. Car A is $100\ m$ behind Car B and is accelerating while car B is moving with a constant velocity. Set up the equations that would determine when Car A catches up to Car B.
13. A train decelerates uniformly from $40ms^{-1}$ to $20ms^{-1}$ over a distance of $200\ m$. Find the acceleration and the time taken.
14. Sketch a velocity-time graph for an object that accelerates from rest, then moves at constant velocity, then decelerates to a stop.
15. On the moon, gravity is $1.6ms^{-2}$. If an astronaut drops a hammer from a height of $2\ m$, how long does it take to hit the ground?

4 Vector in Two Dimensions



We have previously learned how to add vectors in one dimension, where we add their magnitudes when they are in the same direction and subtract them when they are in opposite directions. Now, we will learn how to add vectors in two or three dimensions. For example, if a fishing boat moves in more than one direction, how can we find its total displacement?



We represent the total displacement vector with an arrow that starts from the initial point and ends at the final point over a given period.

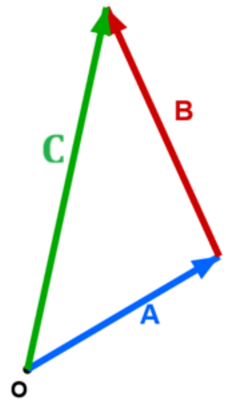
By using an appropriate scale, we can determine the magnitude and direction of the resultant displacement (the sum of displacements). This method of adding vectors is known as the graphical method of vector addition.

4.1 Adding Vectors in two Dimensions

Suppose an object cuts off a displacement $\vec{A}$ and then follows it with another displacement $\vec{B}$

The result will be as if it moved straight from the starting point to the end point which represents the vector C→ (the final displacement). We call the vector $\vec{C}$ the resultant vector or the sum, and the result can be written as follows:

$$\vec{C} = \vec{A} + \vec{B}$$



Note that the plural here is not an algebraic plural.

Direction Signs of Vectors:

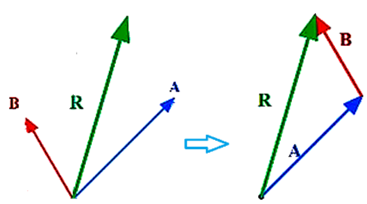
The vector is in the coordinate plane: xy

- positive: when it is in the direction of $+x$ or $+y$
- negative: when it is in the direction of $-x$ or $-y$

4.1.1 Tail to Head Method

Draw the tail of the second vector $\vec{B}$ from the head of the first vector $\vec{A}$,

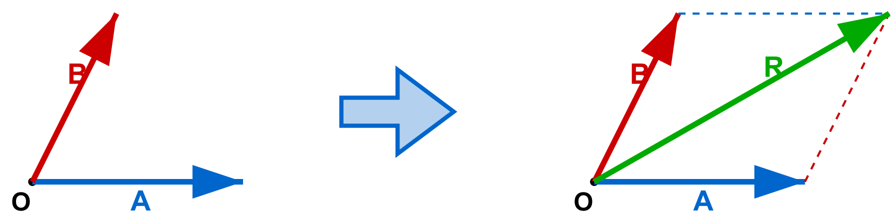
The resultant is $\vec{C}$: a vector from the first tail to the last head.



4.1.2 Parallelogram Method

We draw the two vectors $\vec{A}$ and $\vec{B}$ so that they have a common tail and complete the parallelogram.

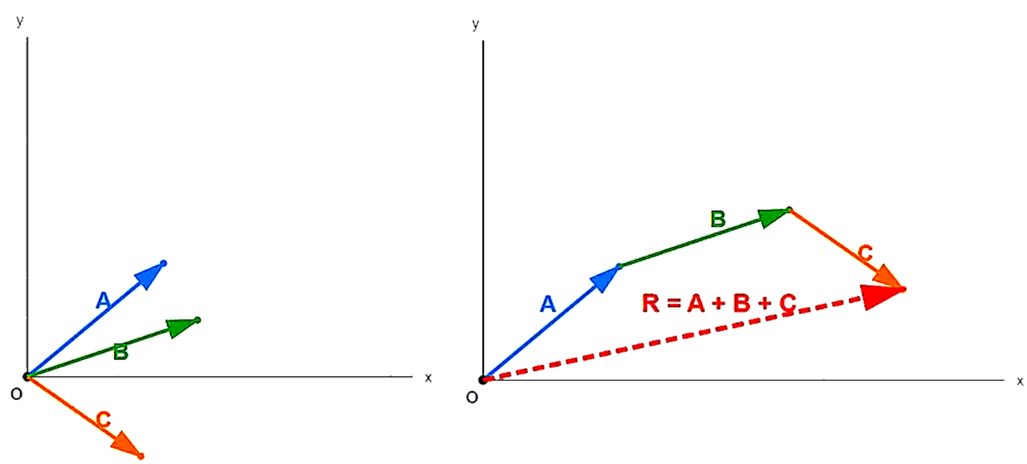
The resultant $\vec{R}$ is the diagonal of a parallelogram with the same tail.



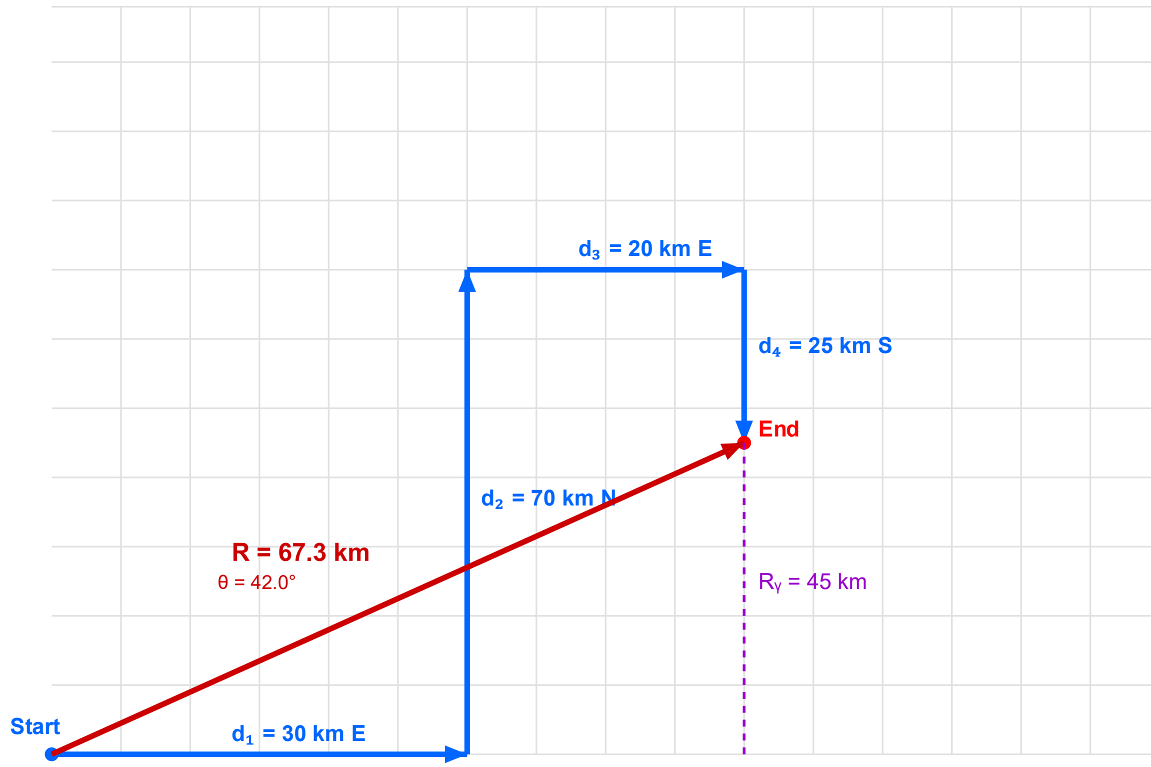
Vector translation: Vectors can be displaced from one position to another while maintaining their magnitude and direction.

4.1.3 Polygon Method

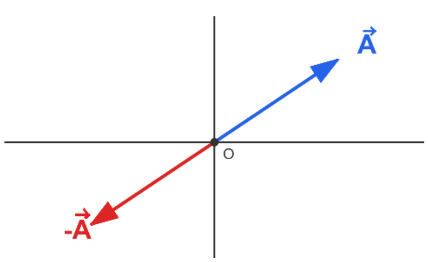
We draw the vectors in succession so that the tail of each vector starts from the head of the previous vector. The resultant $\vec{d}$ is a vector from the first tail to the last head.



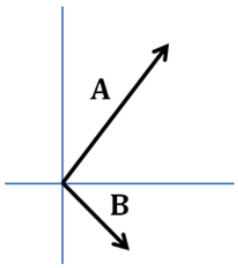
Example 4.1: A car moved 30.0 km east, then turned 70.0 km north, then turned east again and moved 20.0 km, then headed 25.0 km south. Use the coordinate plane to represent the motion of the car and then find the displacement.



Note: The inverse of a vector $-\vec{A}$ (negative of a vector) has the same magnitude as the vector $\vec{A}$ but in the opposite direction of it.



Concept check 4.1: In Fig. draw: $\boldsymbol{A}+\boldsymbol{B}$ and $\boldsymbol{A}-\boldsymbol{B}$



Standard Angle and Reference Angle:

Standard Angle θ: It is measured starting from the $+x$-axis in a counterclockwise direction.

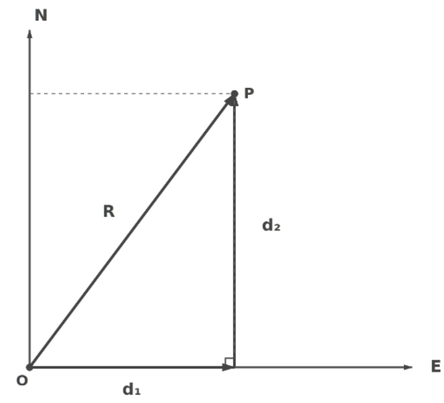
Reference Angle θ': It is measured between the vector and one of the axes.

Translation of Vectors: A vector can be moved from one position to another provided that its magnitude and direction are maintained.

Concept check 4.2: What is the relationship between two vectors $\boldsymbol{A}$ and $\boldsymbol{B}$ IF: $|\boldsymbol{A}+\boldsymbol{B}|=|\boldsymbol{A}-\boldsymbol{B}|$?

Example 4.2: A car travels $3.0\ \mathrm{km}$ east and then $4.0\ \mathrm{km}$ north. Find the resultant displacement.

Solution:



Let the eastward displacement be $\vec{A} = 3.0$ km(east), and the northward displacement be $\vec{B} =$ 4.0 km(north).

Resultant: $\vec{R} = \vec{A} + \vec{B}$.

1) Magnitude

Since the displacements are perpendicular:

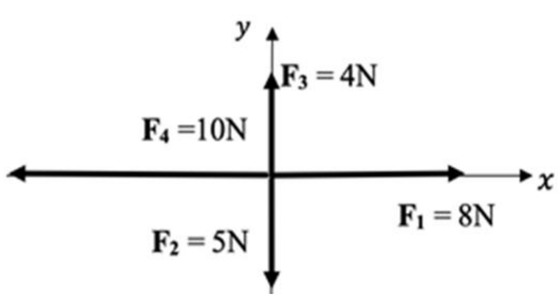
$$R = \sqrt{A^2 + B^2} = \sqrt{(3.0\,\text{km})^2 + (4.0\,\text{km})^2} = 5.0\,\text{km}$$

2) Direction

Angle measured north of east:

$$\theta = \tan^{-1}\left(\frac{B}{A}\right) = \tan^{-1}\left(\frac{4.0\,\text{km}}{3.0\,\text{km}}\right) \approx 53.1^\circ \text{ north of east}$$

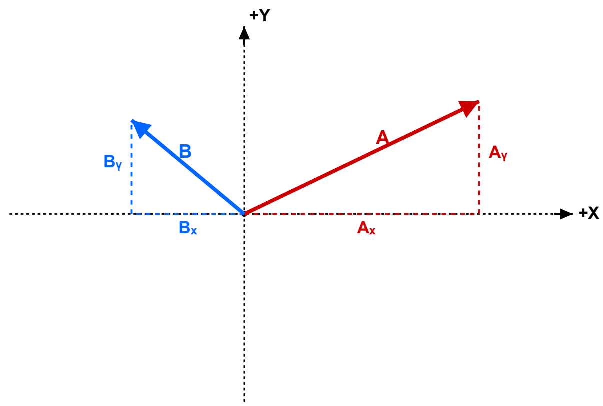
Exercise 4.1: Find the resultant of the coplanar force system shown in Fig



4.1.4 Adding Vectors by Components

To determine the result of vectors by the component method, the following procedure is employed:

Given two vectors $\vec{A}$ and $\vec{B}$, their resultant $\vec{R}$ is shown in the figure.



The resultant $\vec{R}$ has two components given by:

$$\boldsymbol{R_x = A_x + B_x} \qquad \boldsymbol{R_y = A_y + B_y}$$

The magnitude of the resultant: $R = \sqrt{{R_x}^2 + {R_y}^2}$

The direction of the resultant: $tan\,\theta = \frac{R_y}{R_x}$

For a greater number of vectors, this procedure is simply repeated using the same method

Example 4.3: Compute algebraically the resultant of following coplanar displacements: 20.0m at 30.0^o ,40m at 120.0^o , 25.0m at 180.0^o, 42.0m at 270°, and 12m at 315.0°.

Solution:

Let angles be measured counter $-$ clockwise from the $+x$ axis.

Horizontal component:

$$R_x = (20.0\text{ m})cos30° + (40\text{ m})cos120° + (25.0\text{ m})cos180° + (42.0\text{ m})cos270° + (12\text{ m})cos315°$$
$$R_x = -19.1942\text{ m}$$

Vertical component:

$$R_Y = (20.0\ \text{m})sin30° + (40\ \text{m})sin120° + (25.0\ \text{m})sin180° + (42.0\ \text{m})sin270° + (12\ \text{m})sin315°$$

$$R_Y = -5.8443\ m$$

Magnitude

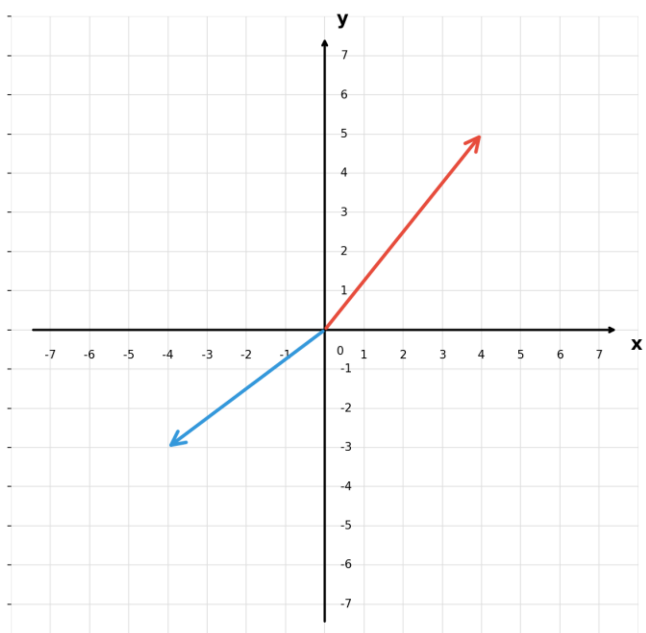
$$|\ \vec{R}\ | = \sqrt{R_x^2 + R_y^2} = \sqrt{(-19.1942\ \text{m})^2 + (-5.8443\ \text{m})^2}$$
$$= \sqrt{368.43\ \text{m}^2 + 34.17\ \text{m}^2} = \sqrt{402.60\ \text{m}^2}$$
$$= 20.06\ \text{m}. = 20.1m$$

Direction

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{-5.8443\ \text{m}}{-19.1942\ \text{m}}\right) = -163.07°.$$

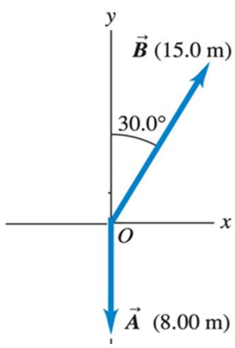
In standard form, $\theta = 196.93°$.

Exercise 4.2: Resolve each vector in the figure, then find the resultant of the two vectors using the component method.



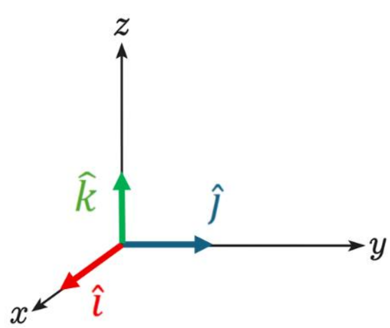
Exercise 4.3: A vector in first quadrant of the plane xy, the value of its component on +x axis is equal 3, and the value of its component on +y axis is equal 6, if the vector rotates clockwise in the first quadrant, and its component in +x axis is multiplied , find the value of its component on +y axis.

Exercise 4.4: find magnitude and direction and draw: $\vec{A} - \vec{B}$



4.1.5 Vector Addition Using the Unit Vector Method

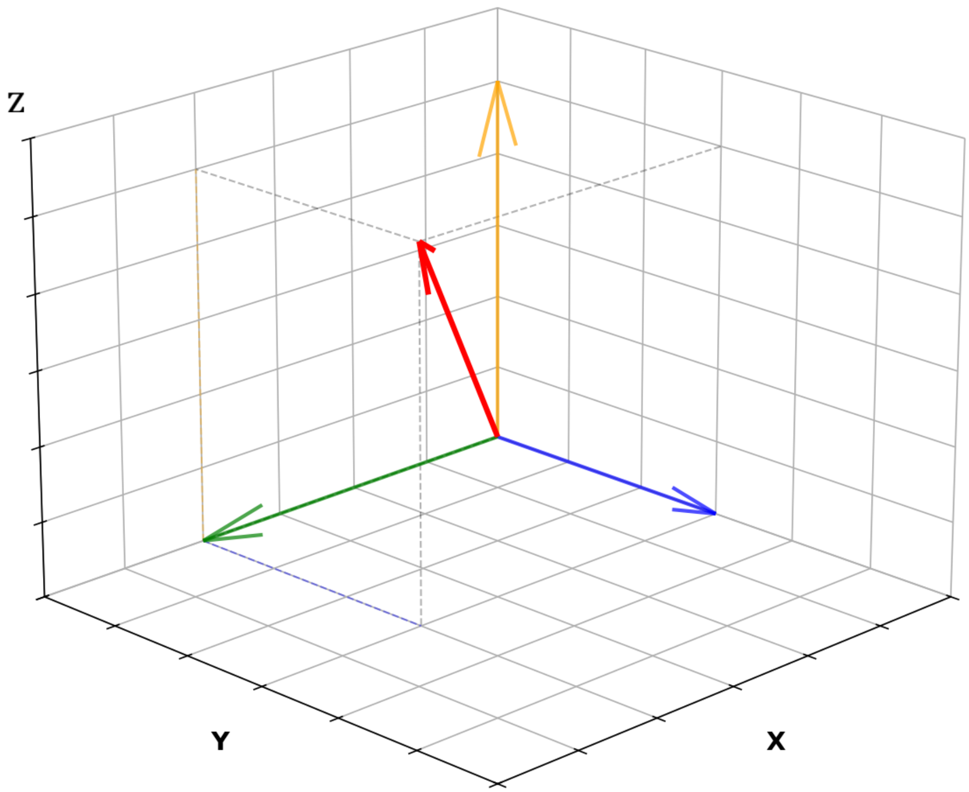
Unit vectors are three vectors $\hat{\imath}$, $\hat{\jmath}$, $\hat{k}$ with magnitudes of 1 and oriented in the directions: $+x$, $+y$ and $+z$ respectively. Any vector can be expressed in terms of unit vectors, which facilitates the handling of vectors when performing vector operations on them. Thus, a unit vector has no magnitude value and is used only to specify direction.



For example, the vector [$\vec{A}$] shown in the figure can be written in terms of unit vectors as follows:

$$\vec{A}_x = A_x\hat{\imath}$$
$$\vec{A}_y = A_y\hat{\jmath}$$
$$\vec{A}_z = A_z\hat{k}$$

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$



Example 4.4: The perpendicular components of the vector acceleration are: $a_x = 6.00\ \text{m/s}^2$, $a_y = 4.00\ \text{m/s}^2$, $a_z = 9.00\ \text{m/s}^2$, Find the vector expression for $\vec{a}$ and its value.

$$\vec{a} = [6.00\hat{\imath} + 4.00\hat{\jmath} + 9.00\hat{k}]\ (\text{m/s}^2)$$

$$|\vec{a}| = a = \sqrt{(6^2 + 4^2 + 9^2)} \approx 11.5\ \text{m/s}^2$$

Example 4.5: A vector $\vec{A}$ has magnitude 4.20 m at an angle $55.0°$above the $+x$-axis. Another vector $\vec{B}$ has magnitude 2.80 m at an angle $-35.0°$(i.e., $35.0°$below the $+x$-axis).

Let $\vec{C} = 2.50\,\vec{A} - 3.20\,\vec{B}$.

(a) Write $\vec{A}$ and $\overrightarrow{B}$ in unit-vector form.

(b) Find $\vec{C}$ in unit-vector form.

(c) Find the magnitude and direction (from $+x$, counterclockwise) of $\vec{C}$.

Solution:

(a) Unit-vector forms

$\vec{A} = (4.20\cos\ 55.0°)\,\hat{\imath} + (4.20\sin\ 55.0°)\,\hat{\jmath} = (2.4090\text{ m})\,\hat{\imath} + (3.4404\text{ m})\,\hat{\jmath}$

$\vec{B} = (2.80\cos\ (-35.0°))\,\hat{\imath} + (2.80\sin\ (-35.0°))\,\hat{\jmath} = (2.2936\text{ m})\,\hat{\imath} + (-1.6060\text{ m})\,\hat{\jmath}$

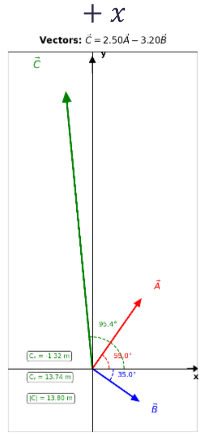
(b) Compute $\vec{C} = 2.50\,\vec{A} - 3.20\,\vec{B}$

$$C_x = 2.50(2.4090) - 3.20(2.2936) = -1.3170\text{ m},$$
$$C_y = 2.50(3.4404) - 3.20(-1.6060) = 13.7403\text{ m}.$$
$$\vec{C} = (-1.317\,\hat{\imath} + 13.740\,\hat{\jmath})\text{ m}$$

(c) Magnitude and direction for $\vec{C}$

$$|\,\vec{C}\,| = \sqrt{C_x^2 + C_y^2} = \sqrt{(-1.317)^2 + (13.740)^2} = 13.803\text{ m} \approx \boxed{13.80\text{ m}}$$
$$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{13.740}{1.317}\right) = 84.5°.$$

This angle is northwest as shown in the drawing, and therefore: $\theta \approx 95.5°$ (counterclockwise from $+x$



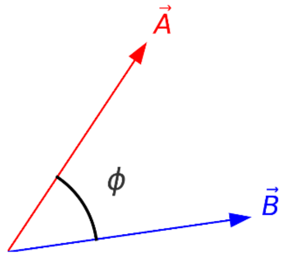
Exercise 4.5: If $\boldsymbol{A} = 2\hat{\imath} - 3\hat{\jmath} + 5\hat{k}$ mm and $\boldsymbol{B} = -\hat{\imath} - 2\hat{\jmath} + 7\hat{k}$ find in component form:

(a) $2\boldsymbol{B} - \boldsymbol{A}$ (b) Vector C such that $\boldsymbol{A} + \boldsymbol{B} + \boldsymbol{C} = 0$

Exercise 4.6: Find the angle between two vectors of equal magnitude: $5.0\ units$, such that the resultant is $[6.0\hat{\jmath}]units$.

4.2 Vector Multiplication

4.2.1 Scalar Product (Dot Product)

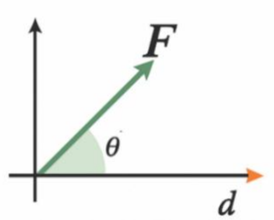


The dot product between two vectors A and B is a scalar quantity given by:

$\boldsymbol{A} \cdot \boldsymbol{B} = |\boldsymbol{A}||\boldsymbol{B}| \cos(\theta)$, where θ is the angle between the vectors.

A physical example: work is the resultant of the dot product of force and displacement.

$$W = Fd\cos\theta$$



Concept check 4.3: How does the angle affect the result of the dot product?

Solution:

- The dot product is positive when 0° < θ < 90°.
- The dot product is zero when θ = 90° (vectors are perpendicular).
- The dot product is negative when 90° < θ < 180°.
- The maximum positive value occurs at θ = 0°.
- The maximum negative value occurs at θ = 180°.

Concept check 4.4: Is the scalar product a commutative operation? And why?

Yes, because multiplication is a commutative operation.

Example 4.6: What is the effect of the angle between two vectors on the value of their dot product?

Solution:

The dot product is positive when the angle between the two vectors is between $0°$ and $90°$.

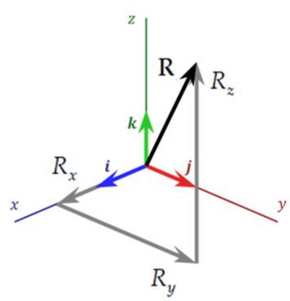
It is zero when the angle is $90°$, meaning the two vectors are perpendicular.

The dot product is negative when the angle is between $90°$ and $180°$.

The dot product reaches its maximum positive value when the angle is $0°$.

It reaches its maximum negative value when the angle is $180°$.

4.2.2 Scalar Product by Unit Vectors



By applying the above relation, we obtain:

$$\hat{\imath}\cdot\hat{\imath} = \hat{\jmath}\cdot\hat{\jmath} = \hat{k}\cdot\hat{k} = (1)(1)\cos 0° = 1$$

$$\hat{\imath}\cdot\hat{\jmath} = \hat{\imath}\cdot\hat{k} = \hat{\jmath}\cdot\hat{k} = (1)(1)\cos 90° = 0$$

Consequently, we can express this as:

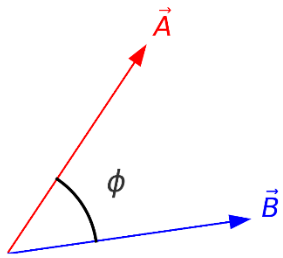
$$\boldsymbol{A}\cdot\boldsymbol{B} = (A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k})\cdot(B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k})$$

$$\boldsymbol{A}\cdot\boldsymbol{B} = A_xB_x + A_yB_y + A_zB_z$$

Exercise 4.7: Force $\vec{F} = (-\hat{\imath} + 2\hat{\jmath})$N acts on an object and displaces it by $\Delta\vec{x} = (2\hat{\imath} + 3\hat{\jmath})$m , calculate:

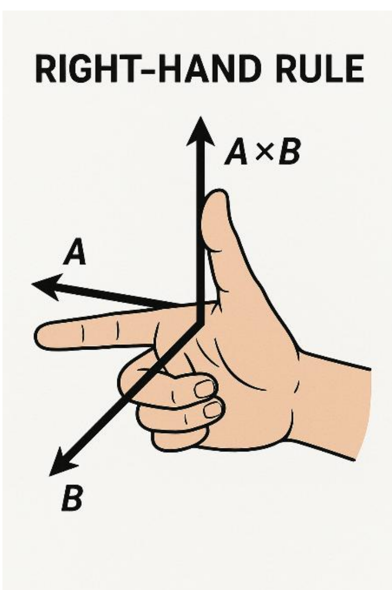
a) Work done by the force. b) Angle between two vectors.

4.2.3 Vector Product (Cross Product)



- yields a vector quantity $|\vec{C}| = |\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}|\, sin\, \phi$
- ϕ The smaller angle between the vectors.
- Direction of the resultant vector: The direction is determined by the right-hand rule and is perpendicular to the plane formed by the two vectors.

The right-hand rule is used to determine the direction of the cross product of two vectors, A × B.



Using your right hand, curl your fingers in the direction from the first vector toward the second vector, your thumb will then point in the direction of the resultant vector ($\vec{C}$).

- Physics Example: Torque

Concept check 4.5: Is the vector product a commutative operation? And why?

4.2.4 Vector Product by Unit Vectors

Note that when applying the above relation

$$\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0$$

$$\hat{\imath} \times \hat{\jmath} = \hat{k}, \qquad \hat{\jmath} \times \hat{\imath} = -\hat{k}$$

$$\hat{\jmath} \times \hat{k} = \hat{\imath}, \qquad \hat{k} \times \hat{\jmath} = -\hat{\imath}$$

$$\hat{k} \times \hat{\imath} = \hat{\jmath}, \qquad \hat{\imath} \times \hat{k} = -\hat{\jmath}$$

Moreover, the cross product can be evaluated using the determinant method

$$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y)\hat{\imath} + (A_z B_x - A_x B_z)\hat{\jmath} + (A_x B_y - A_y B_x)\hat{k}$$

Or: $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

Example 4.8: find the cross product of: $\vec{B} = -3\hat{\jmath}$ $\quad \vec{A} = (-2\hat{\imath} + \hat{k})$

Solution:

Let's write the vectors clearly:

$$\boldsymbol{A} = -2\hat{\imath} + \hat{k} = (-2, 0, 1)$$
$$\boldsymbol{B} = -3\hat{\jmath} = (0, -3, 0)$$

Cross Product $\vec{A} \times \vec{B}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -2 & 0 & 1 \\ 0 & -3 & 0 \end{vmatrix}$$

Compute each component:

- i-component: $(0 \cdot 0) - (1 \cdot -3) = 3$
- j-component: $-((-2 \cdot 0) - (1 \cdot 0)) = 0$
- k-component: $(-2 \cdot -3) - (0 \cdot 0) = 6$

So,

$$\boldsymbol{A} \times \boldsymbol{B} = 3\hat{\imath} + 0\hat{\jmath} + 6\hat{k} = 3\hat{\imath} + 6\hat{k}$$

Exercise 4.8: Vector $\vec{A}$ has magnitude 6 units and is in the direction of the +x axis.

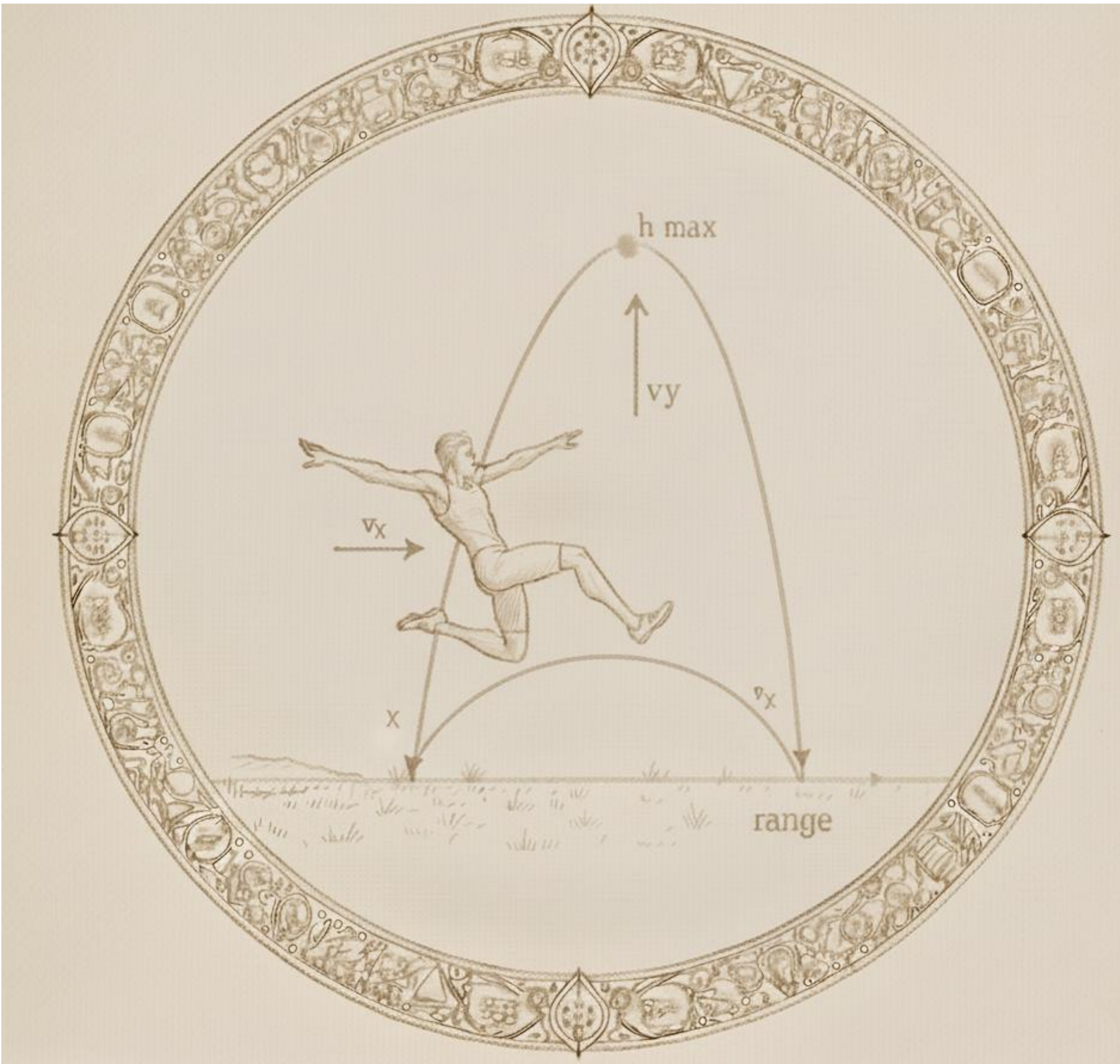
Vector $\vec{B}$ has magnitude 4 units and lies in the xy-plane, making an angle of 30° with the +x axis.

(a) Find the cross product $\vec{C} = \vec{A} \times \vec{B}$

(b) Find components of the vectors $\vec{A}$ and $\vec{B}$

(c) Find the cross product by components. Compare the two results.

5 Motion in Two Dimensions



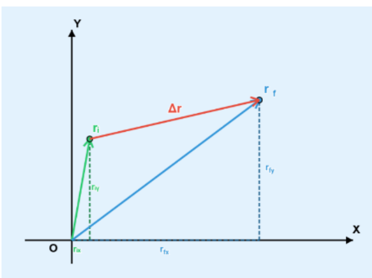
In previous chapters, we described the motion of objects in one dimension, where displacement, velocity, and acceleration all lie along a straight line. However, in many real-life situations, objects move in more than one direction at the same time. For example, when a ball is thrown into the air, it moves both horizontally and vertically, and similarly, a turning car or a boat moving across a river current also undergoes motion in two dimensions.

In this chapter, we will extend our study to motion in two dimensions and learn how to analyze it as two independent motions along the horizontal and vertical directions using vectors. We will apply the concepts of velocity and acceleration in two dimensions to describe projectile motion and uniform circular motion and explore how vectors can be added graphically and algebraically to determine the resultant motion and its direction.

This study represents an important step toward a deeper understanding of the motion of objects in nature and forms the foundation for studying dynamics and Newton's laws, which explain the causes of this motion."

This appears to be from a physics text discussing vector addition and its role in understanding motion and mechanics.

5.1 DISPLACEMENT, VELOCITY, ACCELERATION VECTORS IN MOTION IN TWO DIMENSIONS



We will redefine the motion vectors by considering an object moving between two points A and B. Note that the path need not be a straight line.

The object moves between two positions: [position 1] and [position 2] where the object's positions are defined by position vectors. The displacement of the object (i.e., the straight-line vector between the two positions) is called: The position vectors and displacement can be expressed in terms of unit vectors as follows

$$\vec{r_i} = x_i\hat{\imath} + y_i\hat{\jmath}$$
$$\vec{r_f} = x_f\hat{\imath} + y_f\hat{\jmath}$$
$$\Delta\vec{r} = \vec{r_f} - \vec{r_i} = (x_f - x_i)\hat{\imath} + (y_f - y_i)\hat{\jmath}$$
$$\Delta\vec{r} = \Delta x\hat{\imath} + \Delta y\hat{\jmath}$$

The average velocity vector is defined as the displacement divided by the time interval:

$$\vec{v} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{\imath} + \frac{\Delta y}{\Delta t}\hat{\jmath}$$

Why is it in the direction of $\Delta\vec{r}$? Because it is the vector quantity in the division in the previous equation.

The average acceleration is defined as the change in instantaneous velocity divided by the time interval.

It is in the direction of $\Delta\vec{v}$, Because it is the vector quantity in the division in the previous equation.

Example 5.1: A camel is at the origin of coordinates at time $t_1 = 0$. For the time interval from $t_1 = 0$ to $t_2 = 12.0$ s, the unicorn's average velocity has x component -3.8 m/sand y component 4.9 m/s. At time $t_2 = 12.0$ s.

A) What are the x and y coordinates of the camel?

B) How far is the camel from the origin?

Solution:

A)

$$x_2 = x_1 + v_x(t_2 - t_1) = 0 + (-3.8)(12.0) = -45.6\text{m}$$

$$y_2 = y_1 + v_y(t_2 - t_1) = 0 + (4.9)(12.0) = 58.8\text{m}$$

Therefore, $(x, y) = (-45.6\,m, 58.8\,m)$

B) $r = \sqrt{(x^2 + y^2)} = \sqrt{((-45.6)^2 + (58.8)^2)} = 74.4\text{ m}$

Exercise 5.1: A bird has x- and y-coordinates: (11.1 m ,3.4 m) at time t_1 =0 s and coordinates:(15.3 m ,−0.5 m) at time $t_2 = 3.0\,s$. For this time interval, find: A) The components of the average velocity. B) The magnitude and direction of it.

5.2 Two-Dimensional Motion with Constant Acceleration

To simplify the analysis of this type of motion, we will apply the principle of independence of motion in each dimension. This means that motion along the x-axis is independent of motion along the y-axis, with neither influencing the other. However, they share a common element (time). We will use the kinematic equations and vector mathematics learned in one-dimensional motion and apply them to two-dimensional motion.

Motion Along the x-axis	Motion Along the y-axis
$v_{yf} = v_{yi} + a_y t$	$v_{xf} = v_{xi} + a_x t$

$$\Delta y = v_{yi}t + \frac{1}{2}a_y t^2$$
$$v_{yf}^2 = v_{yi}^2 + 2a_y \Delta y$$
$$\Delta y = \left(\frac{v_{yf} + v_{yi}}{2}\right)t$$
$$\Delta y = v_{yf}t - \frac{1}{2}a_y t^2$$

$$\Delta x = v_{xi}t + \frac{1}{2}a_x t^2$$
$$v_{xf}^2 = v_{xi}^2 + 2a_x \Delta x$$
$$\Delta x = \left(\frac{v_{xf} + v_{xi}}{2}\right)t$$
$$\Delta x = v_{xf}t - \frac{1}{2}a_x t^2$$

Example 5.2: A particle starts from the origin at $t = 0$ with an initial velocity having an x component of 20.0 m/s and a y component of-15 m/s , the particle moves in the xy plane with an x component of acceleration only, given by $a_x = 4.0$ m/s^2

a. Determine the components of the velocity and its value, direction at $t = 5.0$ s
b. Determine the $x$and y coordinates at $t = 5.0$ s
c. Write the particle's displacement in terms of unit vectors and calculate its value and direction.

Solution:

a.

At $t = 5.0s$:

$$v_x = v_{0x} + a_x t = 20.0 + 4.0(5.0) = 40.0\text{m/s}$$
$$v_y = v_{0y} + a_y t = -15.0 + 0 = -15.0\text{m/s}$$
$$\rightarrow \vec{v} = 40\,\hat{\imath} - 15\,\hat{\jmath}\ (\text{m/s})$$
$$|v| = \sqrt{(40^2 + (-15)^2)} = \sqrt{1825} \approx 42.7\ \text{m/s}$$
$$\theta = tan^{-1}(-15/40) \approx -20.6°(\text{below } +x - axis)$$

b.

$$x = v_{0x}t + ½a_x t^2 = 20(5) + ½(4)(5^2) = 150\text{m}$$
$$y = v_{0y}t + ½a_y t^2 = (-15)(5) + 0 = -75\text{m}$$

c.

$$\boldsymbol{r} = 150\,\hat{\imath} - 75\,\hat{\jmath}\ (m)$$

$$|\boldsymbol{r}| = \sqrt{(150^2 + (-75)^2)} = \sqrt{28125} = 75\sqrt{5} \approx 167.7\ m$$

$$\theta = tan^{-1}(-75 / 150) = -26.6° \text{ (below } +x - \text{axis)}$$

Example 5.3: A car is moving on a horizontal surface with an initial velocity: $\boldsymbol{v_i} = (\boldsymbol{3\hat{\imath}} - \boldsymbol{5\hat{\jmath}})$ m/s from the position: $\boldsymbol{r_i} = (\boldsymbol{15\hat{\imath}} - \boldsymbol{7\hat{\jmath}})$ m, if acceleration is:

$\boldsymbol{a} = (\boldsymbol{2\hat{\imath}} + \boldsymbol{4\hat{\jmath}})$ m/s$^{\boldsymbol{2}}$, find after 10 s:

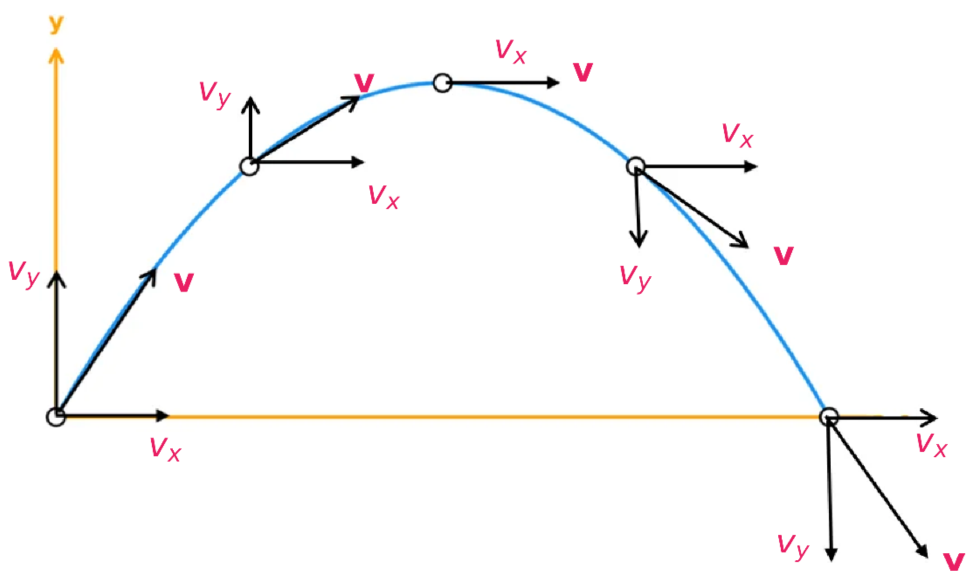
A) Magnitude and direction of the velocity.

B) The Position of the car.

Concept check 5.1: The instantaneous velocity of the particle moving in a circular path centred at the origin is $\vec{v} = (2\text{m/s})\hat{i} - (2\text{m/s})\hat{j}$, through which quadrant is the particle moving at that instant if it is traveling (a) clockwise and (b) counterclockwise around the circle? For both cases. Draw the velocity vector and the position vector.

6 PROJECTILE MOTION

It is a two-dimensional motion in x and y directions, where the projectile follows a parabolic trajectory, such as the motion of a baseball or a football. The only force acting is the gravitational force, and the only acceleration acting is the gravitational acceleration g. Galileo was the first to accurately describe projectile motion, demonstrating that it can be understood by analyzing it into horizontal and vertical components. We shall neglect air resistance in our analysis of projectile motion and assume that it is launched with an initial velocity v_i at an angle θ with the horizontal.



Motion Along the y-Axis:

Vertical acceleration: $\boldsymbol{a_y = -g}$

Initial vertical velocity:

$\boldsymbol{v_{iy} = v_i sin\theta}$

Vertical velocity is variable: it decreases during ascent until it becomes zero at maximum height, then increases during descent.

Vertical velocities are positive during ascent and negative during descent.

Vertical displacement at time t: $\boldsymbol{\Delta y = v_{yi}t - \frac{1}{2}gt^2}$ or $\boldsymbol{\Delta y = \frac{v_{yf}+v_{yi}}{2}t}$

Motion Along the x-Axis:

Horizontal acceleration: $a_x = 0$

Initial horizontal velocity: $v_{0x} = v_0 cos\theta$

Horizontal velocity (constant):

$\boldsymbol{v_x = v_{ix} = v_i cos\theta}$

Horizontal displacement at any instant t:

$\boldsymbol{x(t) = v_{xi}t = (v_i cos\theta)t}$

Horizontal displacement at time t:

$\boldsymbol{\Delta x = v_i cos\theta \cdot t}$

Range: $\boldsymbol{R = v_{xi}}\mathrm{T}$ where T = time of flight (total flight time)

We apply the equations of free fall to calculate vertical motion quantities, where the acceleration is always $\boldsymbol{g}$.

The common element between the horizontal and vertical motions (free fall) is the time of flight $\boldsymbol{t}$

When an object is projected from the top of a cliff with an initial horizontal velocity, it follows a semi-parabolic trajectory.

Guidelines for Problem Solving:

Projectile Motion:

Choose a coordinate system and resolve the initial velocity vector into its x and y components

Follow the methods used for solving constant velocity problems to analyze the horizontal motion and follow the methods for solving constant acceleration problems to analyze the vertical motion.

The motion in both x and y directions share the same time of flight.

Example 6.1: A kicked football leaves the ground at an angle $\theta_o = 37.0^o$ with a velocity of 20.0 m/s, Calculate:

(a) the maximum height, (b) the time of travel before the football hits the ground. (c) horizontal range.

(d) Velocity vector at maximum height.

(e) acceleration vector at maximum height

Assume the ball leaves the foot at ground level and ignore air resistance and rotation of the ball.

Solution:

Given & Components

Initial speed $v_i = 20.0\,\mathrm{m/s}$, launch angle $\theta^0 = 37.0°$, $g = 9.80\mathrm{m/s^2}$.

$$v_{ix} = v_i cos\theta = 15.973\, m/s = 16.0\ \mathrm{m/s}$$

$$v_{iy} = v_i\ sin\theta = 12.036\, m/s = 12.0\ \mathrm{m/s}$$

(a) Maximum Height

$$H_{\max} = \frac{v_{iy}^2}{2g} - 0 = \frac{(12.036)^2}{2 \times (9.80)} = 7.39\ \mathrm{m}$$

(b) Time of Flight

$$T = \frac{2\left(v_{iy} - 0\right)}{g} = \frac{2 \times 12.036}{9.80} = 2.46\ \mathrm{s}$$

(c) Horizontal Range

$$R = v_{ix} \times T = 15.973 \times 2.456 = 39.2\ \mathrm{m}$$

(d) Velocity at Maximum Height

At the top, $v_y = 0$ (instantaneously); v_x is unchanged (no air drag).

$$\boldsymbol{v}_{\mathrm{top}} = 16.0\hat{\imath} + 0.0\hat{\jmath}(\mathrm{m/s}) \rightarrow \text{purely horizontal to } + x$$

(e) Acceleration at Maximum Height

Acceleration is constant and equal to gravity everywhere.

$$\rightarrow \boldsymbol{a} = 0.0\hat{\imath} + -9.80\hat{\jmath}(\mathrm{m/s^2}) \rightarrow \text{downward}$$

Exercise 6.1: A ball is thrown in such a way that its vertical and horizontal components of velocity 40.0 m/sand 20.0 m/s, respectively, estimate the total time of flights and the distance the ball is from its strange point when it lands

Exercise 6.2: Suppose the football in the previous exercise was punted and left the punter's foot at a height of 1.00 m above the ground. How far did the football travel before hitting the ground?

Derive the following equations:

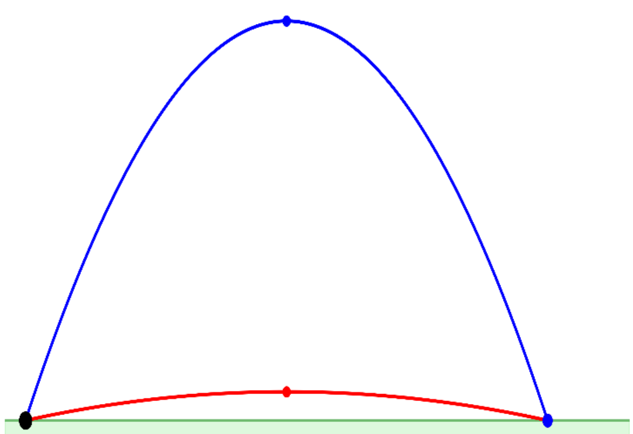
$$h_{max} = \frac{v_i{}^2 \sin^2 \theta_i}{2g} \qquad R = \frac{v_i{}^2 \sin 2\theta_i}{g}$$

Note: (We substitute in both (g) with a positive sign)

Think 6.1: What is the angle that gives the maximum range of the projectile?

Important Note:

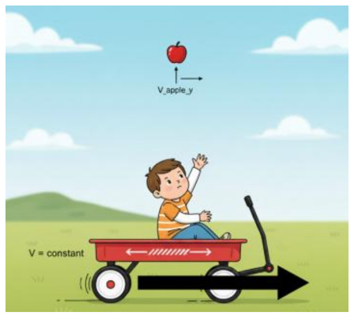
It is observed that the same range can be achieved using the same initial velocity through complementary angles, such as 15° and 75°; however, the maximum height and the total time of flight differ.



Think 6.2: Which flight takes longer, throw with an angle 15° or75^0 ,and why?

Concept check 6.1: You are to launch a rocket, from just above the ground. with one of the following initial velocity vectors: (1)$\vec{v}_i = \mathbf{20}\hat{\imath} + \mathbf{70}\hat{\jmath}$, (2)$\vec{v}_i = -\mathbf{20}\hat{\imath} + \mathbf{70}\hat{\jmath}$, (3)$\vec{v}_i = \mathbf{70}\hat{\imath} + \mathbf{20}\hat{\jmath}$. Rank the vectors according to the time of flight of the projectile. Greatest first.

Concept check 6.2: A child sits upright in a wagon, which is moving to the right at constant speed as shown in Fig. The child extends her hand and throws an apple straight upward (from her own point of view) while the wagon continues to travel forward at constant speed. If air resistance is neglected, will the apple land (a) behind the wagon, (b) in the wagon, or (c) in front of it.



Example 6.2: A stone is thrown from the top of a building upward at an angle $\mathbf{30.0^o}$ of to the horizontal with speed of 20.0 m/s. If the height of the building is 45.0 m.

(a)How long does it take to reach the ground?

(b)What is the speed of the stone just before it strikes the ground?

Solution

Given Data:

$$\text{Initial speed}, v_i = 20.0\ \text{m/s}$$

$$\text{Launch angle}, \theta = 30.0°$$

$$\text{Height of building}, h = 45.0\ \text{m}$$

$$\text{Gravitational acceleration}, g = 9.8\ \text{m/s}^2$$

(a) Time to reach the ground

Using the vertical motion equation:

$$y = v_{iy}t - (1/2)gt^2$$

At the ground: $y = -45.0\text{m}$ (taking the top of the building as $y_0 = 0$)

$$v_{iy} = v_i \, sin(30°) = 20 \times 0.5 = 10.0 \text{ m/s}$$

Substitute: $-45.0 = 10.0t - 4.9t^2$

$$\Rightarrow 4.9t^2 - 10.0t - 45.0 = 0$$

Using the quadratic formula:

$$t = 10 \pm \sqrt{\frac{10^2 + (4 \times 4.9 \times 45)}{2 \times 4.9}}$$

$t = 4.23s$ (neglecting negative root)

(b) Speed just before striking the ground

Vertical velocity: $v_{fy} = v_{iy} - gt = 10.0 - 9.8 \times 4.23 = -31.45 \text{m/s}$ (downward)

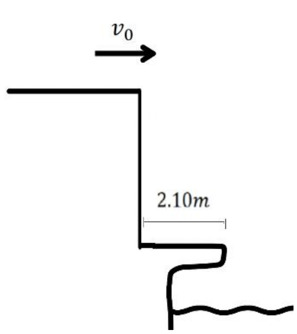
$$Horizontal\ velocity\ (constant): v_x = v_i \, cos(30°) = 20 \times 0.866 = 17.32 \text{ m/s}$$

Total speed: $v = \sqrt{(v_x{}^2 + v_{fy}^2)} = \sqrt{(17.32^2 + 31.45^2)} = 35.9 \text{m/s}$

Exercise 6.3: One of Galileo's conclusions is that a particle that is thrown horizontally reaches the ground at the same time as a particle that is freely falling down. How do you prove it?

Concept check 6.3: If two balls roll on a table's top at two different speeds, and then fall off the table's top at the same time, which one will hit the ground first?

Exercise 6.4: A swimmer dives off a cliff with a running horizontal leap, as shown in Figure. What must his minimum speed be just as he leaves the top of the cliff so that she will miss the ledge at the bottom, which is $\mathbf{2.10\ m}$ wide and $\mathbf{8.00\ m}$ below the top of the cliff?



Exercise 6.5:

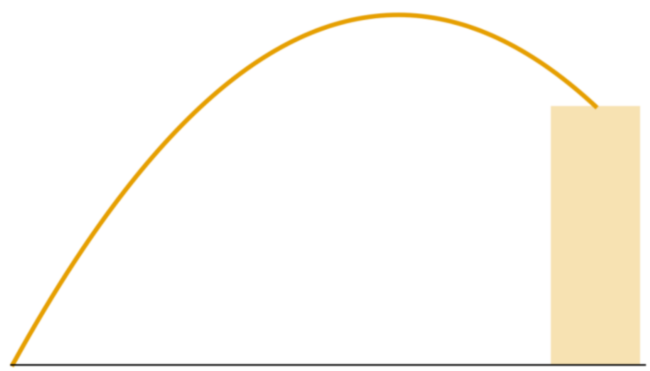
A small projectile is launched from street level toward a tall building with an initial speed of $38.0\ m/s$ at an angle $\theta_0 = 55.0°$ above the horizontal. The stone lands on the flat roof, $4.80\ s$ after launch. Neglect air resistance.

Find:

(a) the roof height above the street,

(b) the speed of the stone just before it touches the roof.

(c) the maximum height the stone reaches above the street.

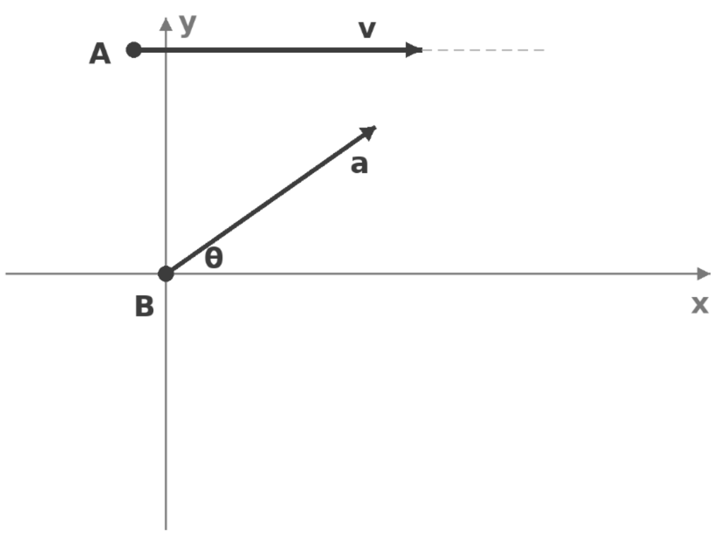


Exercise 6.6: A basketball player stands horizontally 8.0m away from the basket. If he throws the ball at an angle 35^0above the horizontal from a height 2.2m, what initial velocity is needed for the ball to pass through the basket hoop at the height 3.0m directly?

Exercise 6.7: A drone is moving horizontally at a constant speed of $3.0\ \text{m/s}$ at an altitude of $30.0\ \text{m}$ above the ground. If a missile defense platform launches a missile to intercept the aircraft at the instant the aircraft passes directly over the platform, at what angl θ e from the vertical $(y+)$ must the missile be launched to hit the aircraft if the missile's acceleration magnitude is constant in any direction 0.40m/s^2 ?

6.1 ADDITIONAL PROBLEMS

Question 1: A drone moves horizontally at a constant speed of 3.0 m/s at an altitude of 30.0 m above the ground. If defense systems launch a drone moving with constant acceleration in a straight line to intercept the airplane at the moment the airplane passed over the drone platform, what is the angle θ with the vertical (y +) that the drone must be launched at to hit the airplane if the drone starts from rest and its absolute acceleration (in any direction) is constant with a magnitude of 0.40 m/s²?



Question 2: A jet plane is flying at a constant altitude. At time $t_1 = 0$ it has components of velocity: $v_{xi} = 50$ m/sand $v_{yi} = 80$ m/s. At $t_2 = 30.0$ s the components are $v_{xf} = -80$ m/s and $v_{yf} = 40$ m/s.

(a) Sketch the velocity vectors at t_1 and t_2.

(b) Find the components of the average acceleration.

(c) Find the magnitude and direction of the average acceleration.

Question 3: A particle starts moving from the point of origin with an initial velocity $(\mathbf{8.0\hat{j}})$ m/s and moves along $\boldsymbol{xy}$ plane with constant acceleration: $(\mathbf{4\hat{i} + 2\hat{j}})$ m/s^2 , At the moment when the x-coordinate of the particle is $\boldsymbol{x_f = 30}$ m, what is the value of the coordinate $\boldsymbol{y_f}$?

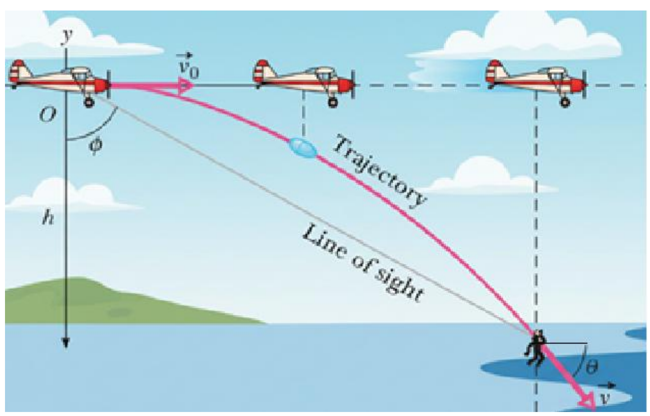
Question 4: At $\boldsymbol{t = 0}$, a particle moving in the $\boldsymbol{xy}$ plane with constant acceleration has a velocity of $\boldsymbol{v_i = (3.00i - 2.00j)}$ m/s and is at the origin. At$\boldsymbol{t = 3.00}$ sthe particle's velocity is$\boldsymbol{v_f =}$ $\boldsymbol{(9.00i + 7.00j)}$ m/s. Find (a) the acceleration of the particle and (b) its coordinates at any time $\boldsymbol{t}$?

Question 5: A hostile ship is located 450 m from a coastal cannon at sea level. The cannon fires cannonballs with an initial speed: $v_0 = 75$ m/s.

(a) At what angle θ_0 from the horizontal must the cannon be fired to hit the ship?

(b) What is the maximum range of cannonballs?

Question 6: a rescue plane flies at 62.0 m/sand constant height $h = 6.00 \times 10^2$ m toward a point directly over a victim, where a rescue capsule is to land. (a)What should be the angle ϕ of the pilot's line of sight to the victim when the capsule release is made? (b) As the capsule reaches the water. What is its velocity in unit-vector notation and magnitude-angle?

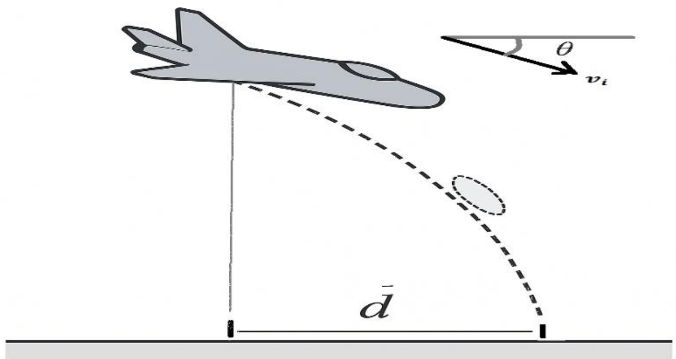


Question 7:

An airplane has a speed of $v_0 = 320$ km/h and is diving at an angle of $\theta = 35°$ below the horizontal when the pilot releases a radar decoy. The horizontal distance between the release point and the point where the decoy strikes the ground is $d = 850$ m.

(a) How long is the decoy in the air?

(b) How high was the release point?



Question 8: A firefighter directs a stream of water from a hose that is elevated $1.20\ m$ above the ground. The water leaves the nozzle at a speed of $24.0\ m/s$ at an angle of $35°$ above the horizontal. A building wall is located $10.0\ m$ horizontally from the nozzle. At what height above the ground will the water strike the wall?

Question 9: Two projectiles are fired at the same time at different angles What is the ratio of their initial velocities if they travel the same horizontal distance at the same time (during the motion)?

Question 10: A boy can throw a ball at a maximum horizontal distance of $\mathbf{40.0\ m}$ on a level field. How far can he throw the same ball vertically upward? Assume that his muscles give the ball the same speed in each case.

7 SIMULATION TEST

Q1. Which of the following is a reasonable size for a grain of sand?

A) $1m$
B) $1cm$
C) $1\text{m}m$
D) $1\mu m$

Q2. The electron volt (eV) is an energy unit used when dealing with microscopic systems. 1 eV is equivalent to 1.6×10^{-19}J. The energy needed to ionize a hydrogen atom is 2.18×10^{-18}J. Express this energy in eV.

A. $73.4\ eV$
B. $13.6\ eV$
C. $3.5\times10^{-35}\text{eV}$
D. $0.1\ eV$

Q3. $V_0 = 270$ ml of water is poured into a glass jar and then frozen. What should be the minimum volume of the jar V so that it does not burst when the water turns into ice? The density of water is $1.00\ \text{g/cm}^3$, and the density of ice is $0.917\ \text{g/cm}^3$.

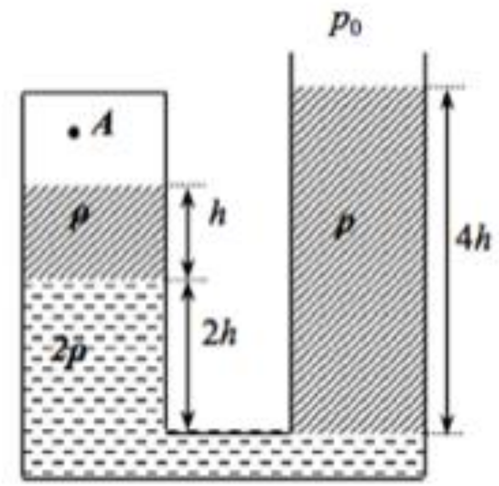
A) $247\ ml$
B) $270\ ml$
C) $294\ ml$
D) $1000\ ml$

Q4. An object floats with $1/4$ part of its volume outside the liquid when put in liquid A and $1/2$ part of its volume outside the liquid when put in liquid B. The density of liquid A is ρ_A. Find the density of liquid B.

A) $\rho_B = \frac{1}{2}\rho_A$
B) $\rho_B = 2\rho_A$
C) $\rho_B = \frac{3}{2}\rho_A$
D) $\rho_B = \frac{2}{3}\rho_A$

Q5. Determine the air pressure above the surface of the liquid at point A inside the closed section of the curved tube. The values are $\rho = 800\ \mathrm{kg/m^3}, h = 20\ \mathrm{cm}, g = 10\ \mathrm{m/s^2}$. Liquids with densities ρ and 2ρ do not mix with each other. The air pressure above the vessel is $p_0 = 10.0\ \mathrm{kPa}$.

A) $24.4\ \mathrm{kPa}$
B) $8.4\ \mathrm{kPa}$
C) $3.6\ \mathrm{kPa}$
D) $11.6\ \mathrm{kPa}$



Q6. An ice cube of pure water floats in a glass of salt water. How will the water level in the glass change after the ice melts?

A) water level increase
B) water level decrease
C) water level remains the same
D) it depends on ice cube volume

Q7. When measuring the pressure in the sea, the following results were found. The pressure at a distance of $50\ \mathrm{m}$ from the bottom is 3times greater than the pressure at a depth of $50\ \mathrm{m}$. Find the depth of the sea. Neglect atmospheric pressure.

A) $100\ m$
B) $200\ m$
C) $450\ m$
D) $650\ m$

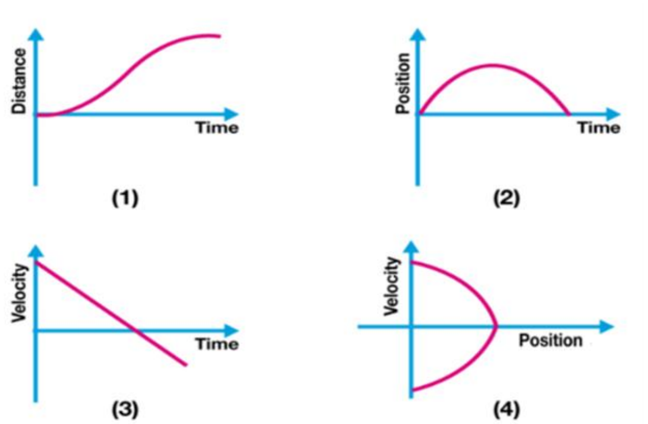
Q8. All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up.

A) 1

B) 2

C) 3

D) 4



Q9. The bus travels a distance of $8\ km$ in 23 minutes. The bus speed is $40\ km/h$ when it moves. How much time does this bus spend at stops?

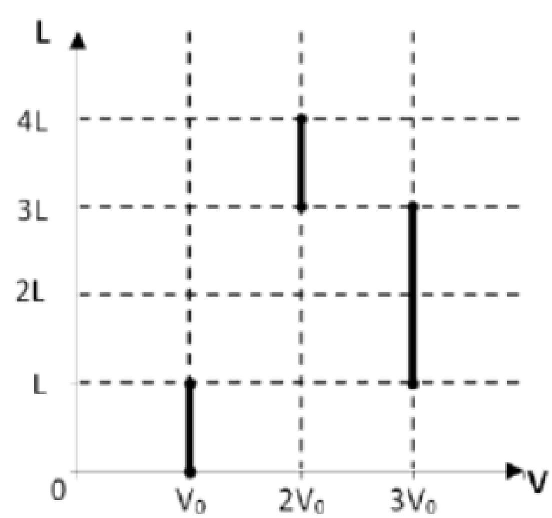
A) 3 minutes

B) 11 minutes

C) 12 minutes

D) 1/5 hour

Q10. The graph shows the dependence of the distance traveled by the body on its speed. Determine the average speed of the body along the entire path. Assume v_0 is given.



A) $\frac{24}{13}\ v_0$

B) $\frac{4}{3}\ v_0$

C) v_0

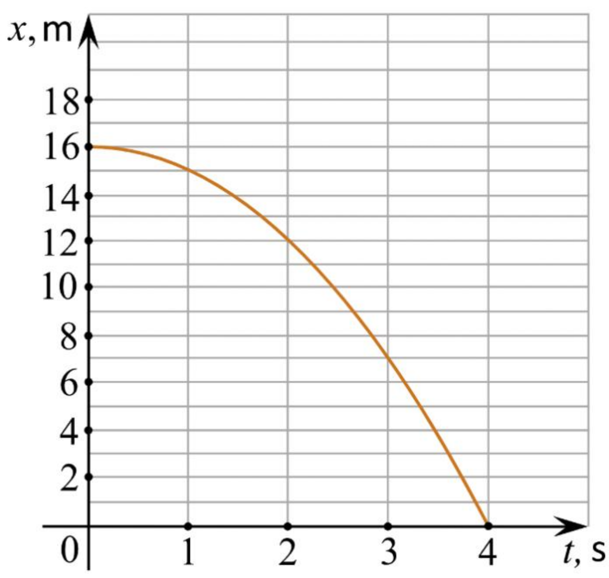
D) $\frac{3}{4}\ v_0$

Q11. Two old men left from A to B and from B to A at sunrise, heading towards one another (along the same road). They met at 12:00, but did not stop, and each of them carried on walking with the same speed. The first man came (to B) at 16:00, and the second (to A) at 21:00. What time was the sunrise that day?

A) 5:00
B) 5:30
C) 6:00
D) 8:00

Q12. A small body starts motion with constant acceleration along the x-axis. Initial velocity is zero. The graph represents the dependence of the coordinate on time. What is the projection of the velocity of this body at time 3s?

A. 0 m/s
B. 6 m/s
C. -6 m/s
D. -2 m/s



Q13. A person standing on the edge of a fire escape simultaneously launches two stones, one straight up with a speed of $7\ \mathrm{m}/s$ and the other straight down at the same speed. How far apart are the two stones $2s$ after they were thrown, assuming that neither has hit the ground?

A. $14\ \mathrm{m}$
B. $20\ \mathrm{m}$
C. $28\ \mathrm{m}$
D. $34\ \mathrm{m}$

Q14. A projectile is thrown upward with speed v. By the time its speed has decreased to $v/2$, it has risen a height h. Neglecting air resistance, what is the maximum height reached by the projectile?

A. $5h/4$
B. $4h/3$
C. $3h/2$
D. $2h$

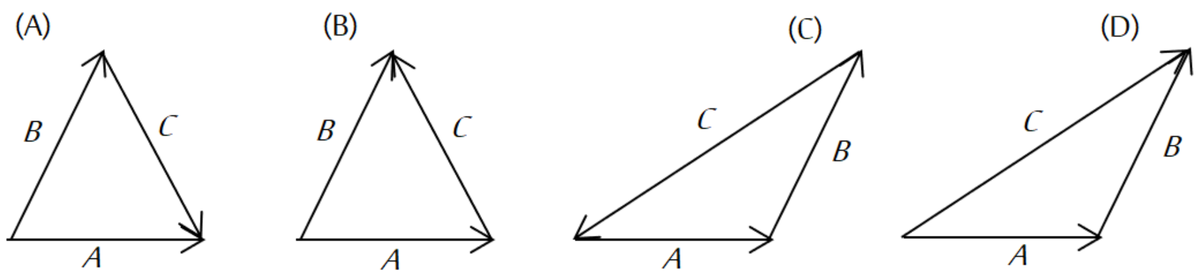
Q15. A car travelling due north at $20\ \mathrm{km/h}$ turns east and then continues to move with the same speed. If it takes $10s$ to complete the turn, the magnitude of the average acceleration during the turn is equal to:

A. 0
B. $1.1\mathrm{m/s^2}$
C. $0.55\ \mathrm{m/s^2}$
D. $0.78\mathrm{m/s^2}$

Q16.If $\vec{A} = \vec{B} + \vec{C}$ and the magnitude of $\vec{A}$, $\vec{B}$ and $\vec{C}$ are 5, 4 and 3 units respectively, then the angle between $\vec{A}$ and $\vec{C}$ is:

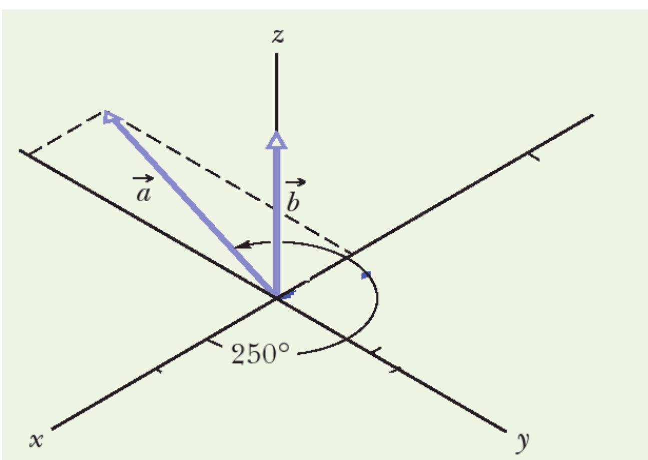
A. 90^0
B. 53.1^0
C. 36.9^0
D. 27.6^0

Q17.Three vectors related by the relation $\vec{C} = -\vec{A} - \vec{B}$. Which of the following diagrams represents the correct relationship between them?



Q18. In the figure, vector $\vec{a}$ lies in the xy plane, has a magnitude of 18 units and points in direction 250^0 from the positive direction of the x-axis. Also, vector $\vec{b}$ has a magnitude of 12 units and points in the positive direction of the z-axis.

if: $\vec{c} = \vec{a} \times \vec{b}$, The angle that vector $\vec{c}$ makes with the x-axis is equal to:



A. 210^0
B. 160^0
C. 90^0
D. 0

Q19.The range of a projectile launched at an angle of 15^0 to the horizontal is 1.5 km. What will be its range if it is projected at an angle of 45^0 to the horizontal with the same total velocity?

A. 0.75 km
B. 3.0 km

C. $1.5\ \text{km}$
D. $6.0\ \text{km}$

Q20.The speed of a projectile at the maximum height is half of its initial speed u

Its horizontal range is:

A. $\frac{u^2}{\sqrt{3}g}$
B. $\frac{2u^2}{\sqrt{3}g}$
C. $\frac{\sqrt{3}u^2}{2g}$
D. $\frac{\sqrt{3}u^2}{g}$

Q21. A grasshopper can jump a maximum horizontal distance of $0.2\ m$. If it continues to jump in this manner, spending negligible time on the ground, then the speed with which he moves forward is approximately:

A. $1\ \text{m/s}$
B. $2\ \text{m/s}$
C. $3\ \text{m/s}$
D. $4\ \text{m/s}$

Q22.Two objects, A and B, are falling freely towards the Earth while parallel to each other. If horizontal acceleration is added to object A, which of the following statements is true? Neglect air resistance.

A. Object A reaches the Earth's surface first
B. Object B reaches the Earth's surface first
C. Both objects reach the Earth's surface together
D. The vertical acceleration of object A increases

Q23. A particle is moving on a horizontal surface with an initial velocity: $[-\vec{\imath} + 3\,\vec{\jmath}\,]\text{m}/s$ from the position $[2\,\vec{\imath} - 5\,\vec{\jmath}\,]\text{m}$, if acceleration is: $[\vec{\imath} + 2\,\vec{\jmath}\,]\text{m/s}^2$, its average velocity after 5 seconds in (m/s) is:

A. $4\,\vec{\imath} + 13\,\vec{\jmath}$
B. $9.5\,\vec{\imath} + 35\,\vec{\jmath}$

C. $7.5\vec{i} + 40\,\vec{j}$

D. $5\,\vec{i} + 15\,\vec{j}$

Q24. Which of the following graphs correctly represents the change in the vertical component v_y of the velocity of a projectile with time t, until it returns to the Earth's surface, given that it was launched from the Earth's surface at an angle of 45° with the horizontal?