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Ministry of Education

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National Science and Mathematics Olympiad

Learning Materials for the Physics Track

National Teams Competition 2026

Physics

CONTENTS

Introduction

Up to this point, the study of motion has been framed primarily through the analysis of the forces acting on bodies. An alternative and often more general formulation involves describing motion in terms of the quantity's energy (the subject of this chapter) and momentum (to be discussed in the following chapter). The fundamental significance of these quantities lies in their conservation within certain physical systems. That is, under a wide range of conditions, their total value remains invariant. The existence of such conserved quantities provides powerful analytical tools for addressing complex physical problems.

The principles of conservation of energy and momentum are particularly indispensable in the study of systems composed of many interacting particles, where a detailed force-based analysis becomes impractical or even impossible. These conservation laws possess remarkable generality, applying across a vast spectrum of physical phenomena—from macroscopic mechanical systems to atomic and subatomic domains—where Newtonian mechanics alone ceases to be sufficient.

This chapter is devoted to an in-depth exploration of the concept of energy and its intimately related quantity, work. Both energy and work are scalar quantities, characterized solely by magnitude and devoid of directional dependence. This scalar nature often renders them more tractable in analysis than vector quantities such as force and acceleration, while still capturing the essential physics governing the behavior and transformations of mechanical systems.

chaptre1 WORK AND ENERGY

1.1 WORK

In everyday language, the term work can have several meanings, but in physics it carries a precise and specific definition. In physical terms, work describes what is accomplished when a force acts on an object and causes it to move through a displacement.

For clarity, we shall consider only translational motion, assuming that the object behaves as a rigid body without internal deformation and can thus be treated as a single particle. Under these assumptions,

1.1.1 Work Done by A Constant Force

The work done on an object by a constant force, that is, a force that remains constant in both magnitude and direction is defined as the product of the magnitude of the displacement and the component of the force that acts parallel to the displacement.

In mathematical form, this relationship is expressed as:

$W=F_{\parallel} d$

where $F_{\parallel}$ is the component of the constant force F that lies parallel to the displacement d.

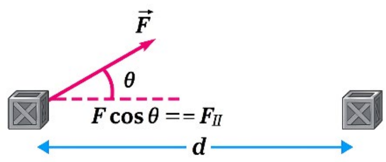
Alternatively, it may also be written as:

$$W = f\ d\cos\theta \qquad (1)$$

Here, F is the magnitude of the applied force, d is the magnitude of the displacement, and θ is the angle between the directions of the force and the displacement. The cosine factor ensures that only the portion of the force acting in the direction of motion contributes to the work done.

Work is a scalar quantity it possesses magnitude but no direction. It can, however, be positive or negative, depending on whether the force component acts in the same direction as the displacement or in the opposite direction.

When dealing with work and forces, it is crucial to specify which force is performing the work and on which object. In systems where multiple forces act, one must distinguish between the work done by each individual force and the net work performed by the total (resultant) force on the object

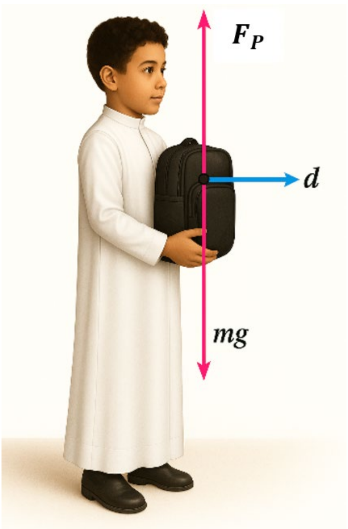


Concept Check

A student lifts a box of mass m vertically h with constant velocity and then walks horizontally $\vec{d}$. What is:

-The total work done by the student,

- the work done by force$\vec{F}_P$?



Important Notes

-Work is a scalar quantity, it has no direction, but only magnitude, which can be positive or negative.

-Work is a means of transferring energy to and from a particle.

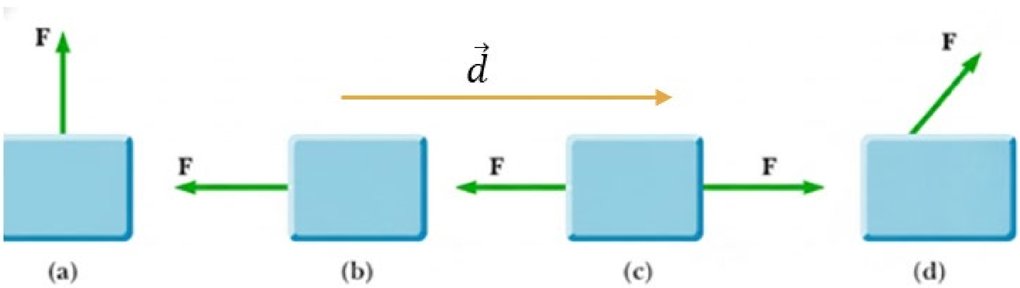
Work is positive: if the force has a component in the direction of motion, and it increases particle's velocity (its kinetic energy), and an Energy is transmitted to the particle.

Work is negative: if the force has a component in opposite direction of motion, and it decreases particle's velocity (its kinetic energy), and an Energy is transmitted from the particle

Think:

Figure shows four situations in which a force is applied to an object. In all four cases, the force has the same magnitude, and the displacement of the object is to the right and of the same magnitude. Rank the situations in order of the work done by the force on the object, from most positive to most negative.

Note that positive work means energy is transferred to the object and vice versa

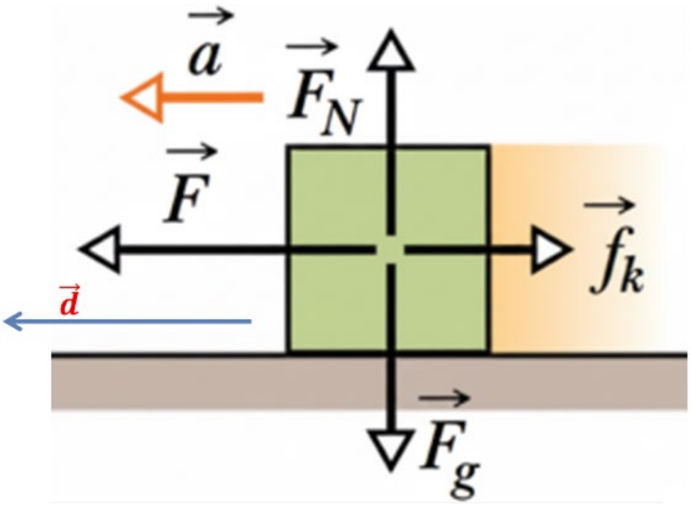


1.1.2 Work Done by the Kinetic Friction Force

The work done by the kinetic friction force is negative always, because its direction is opposite to the direction of displacement. We calculate it by the equation

$$W_k = -f_k d \qquad (2)$$

Where d is the length of the friction path, regardless of its shape (straight or otherwise)



Problem –Solving Hints

1. Draw a free-body diagram showing all the forces acting on the object you choose to study.
2. Choose an xy coordinate system. If the object is in motion, it may be convenient to choose one of the coordinate directions as the direction of one of the forces, or as the direction of motion. [Thus, for an object on an incline, you might choose one coordinate axis to be parallel to the incline.]

3. Apply Newton's laws to determine unknown forces.

4. Find the work done by a specific force on the object by using $W = Fd\cos\ \theta$ for a constant force. The work done is negative when a force opposes the displacement.

5. To find the net work done on the object, either:

(a) find the work done by each force and add the results algebraically; or

(b) find the net force on the object F_{net}, and then use it to find the net work done, which for constant net force is: $W = Fd\cos\ \theta$

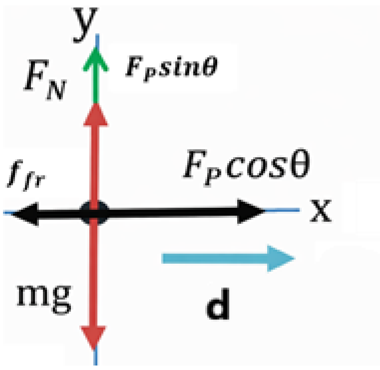
Example 1.1

A man pulls a box of mass 50 kg a distance of 40 m along a horizontal surface with a constant force of 100 N applied at an angle of 37° above the horizontal, as shown in the figure. The surface is rough and exerts a frictional force of 50 N on the box.

(a) Calculate the work done by each force on the crate.

(b) Determine the net work done on the crate.

Solution:



work done by the gravitational force mg and the normal force F_N is zero, now the two forces are perpendicular to the displacement (cos90=0).

$$W_g = 0 \quad W_N = 0$$

work done by F_P $\quad W_P = F_P d cos\theta = 3.2 \times 10^3 J$

W_{fr} = the work done by F_{fr} $\quad F_{fr} d cos180 = -1.2 \times 10^3 J$

The net work

$$W_{net} = W_g + W_N + W_P + W_{fr} = 1.2 \times 10^3 J$$

Or by the net force

Where the displacement in the vertical direction is zero, then the work is only due to the horizontal forces.

$$F_{net(x)} = F_P d cos\theta + F_{fr} d cos180 = 30N$$
$$W_{net} = F_{net(x)} \mathrm{d} = 30N \times 40m = 1.2 \times 10^3 J$$

Exercise1.1

A body of mass $m = 20.0$ kg moves on a rough horizontal surface with a constant velocity under the action of a constant force $\vec{F}$ that makes an angle of $\theta = 37.0°$ above the horizontal. If the coefficient of kinetic friction between the body and the surface is $\mu_k = 0.40$, calculate the work done by the force when the body undergoes a displacement of $d = 8.50$ m

Exercise1.2

During a windstorm, a smooth box slides over an oily patch such that its displacement is given by:

$$\vec{d} = (-3.0\,\mathrm{m})\,\hat{\imath}$$

A constant wind force acts on it, given by:

$$\vec{F} = (2.0\,\mathrm{N})\,\hat{\imath} + (-6.0\,\mathrm{N})\,\hat{\jmath}$$

Calculate the following:

1- The magnitude of the force, The magnitude of the displacement, The angle between the force and displacement vectors.

2- The work done by the force on the box during the displacement.

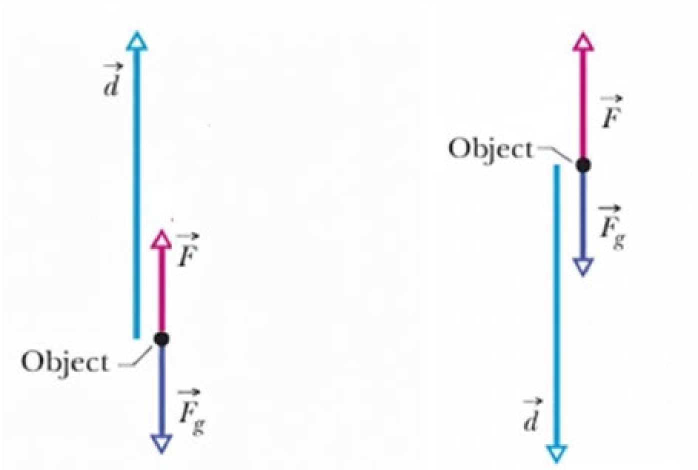
1.1.3 Work Done by The Gravitational Force

The figure shows two cases of particle motion:

a) An applied force lifts the object. The object's displacement makes an angle of $180°$ with the gravitational force.

b) An applied force acts on the object; The displacement of the object makes an angle $0°$ with the gravitational force.

Explain the positive and negative signs of work done by the applied force $\vec{F}$ and the gravitational force $\vec{F_g}$.



An object of mass m $m$oves perpendicular to the earth's surface

If the body is rising: the work done by the force of gravity is negative:

$$W_g = mgdcos(180) \qquad (3)$$

If the body is rising: the work done by the force of gravity is positive.

$$W_g = mgdcos(0) \qquad (4)$$

Example1.2

Work on a Backpack Aman carries a backpack of mass $m = 15.0$ kg up a hill to a vertical height of $h = 10.0$ m, as shown in the figure. Assume the motion is smooth and occurs at a constant velocity ($a = 0$). Calculate:

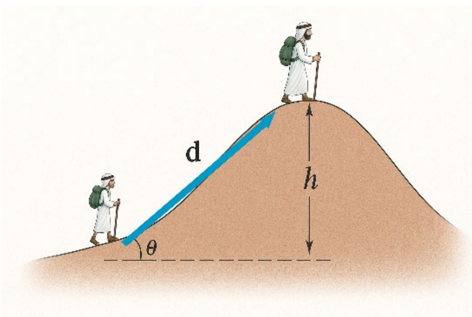
(a) the work done by the man on the backpack W_{man},

(b) the work done by the gravitational force W_g, and

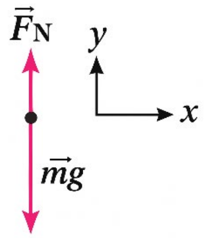
(c) the net work done on the backpack W_{net}.

Use $g = 9.80$ m/s^2, and express all numerical results with the correct number of significant figures

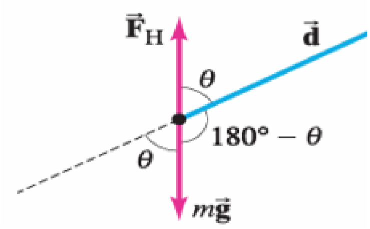
Solution:



Draw a free-body diagram.



Choose a coordinate system



(a) Work done by the man on the backpack W_{man}

The applied force acts upward, in the same direction as the displacement:

$$W_{\text{man}} = F\,h \cos\,0° = Fh$$

Since $F = mg$:

$$W_{\text{man}} = (15.0)(9.80)(10.0) = 1470 \text{ J}$$

Considering three significant figures:

$$W_{\text{man}} = 1.47 \times 10^3 \text{ J}$$

(b) Work done by the gravitational force W_g

The gravitational force acts downward, opposite to the direction of displacement, so the angle $\theta = 180°$:

$$W_g = F_g\, h \cos 180° = -mgh$$
$$W_g = -(15.0)(9.80)(10.0) = -1470 \text{ J}$$

With correct significant figures:

$$W_g = -1.47 \times 10^3 \text{ J}$$

(c) Net work done on the backpack W_{net}

$$W_{\text{net}} = W_{\text{man}} + W_g$$
$$W_{\text{net}} = (1.47 \times 10^3) + (-1.47 \times 10^3) = 0$$

Conclusion:

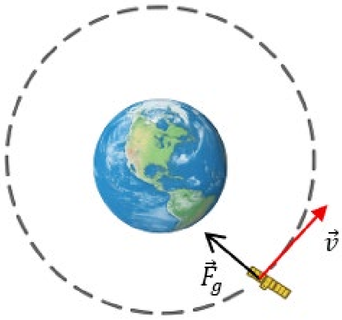
Because the backpack moves at a constant velocity (zero acceleration), the positive work done l

Top of Form

Bottom of Form

Concept Check

What is the work done by the gravitational force that the Earth exerts on the satellite, keeping the satellite in its circular path?



Note:

This is why the Moon, as well as artificial satellites, can stay in orbit without expenditure of fuel: no work needs to be done against the force of gravity

1.1.4 Work Done by A Varying Force

The work done on an object can be calculated using the force-dependent formula: $W = F\ d\ cos\theta$. However, the basic idea of work remains valid even when the force is changing in quantity or direction.

However, in many cases, it does not remain for long, but rather degrades as the object moves, either in its consumption or in its direction. For example:

• The gravitational force acting on a rocket decreases the further it moves away from Earth, because it is inversely proportional to the square of the distance from the center of the Earth.

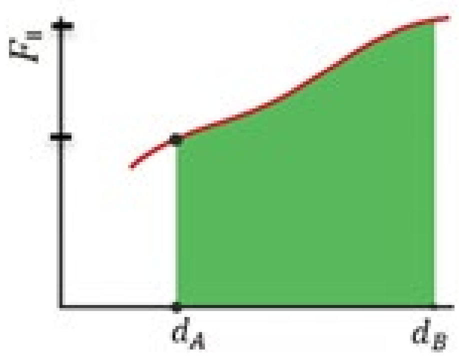
• The force of spring increases the more elongated it is.

• The force also changes when a cart or box is pushed down a slope.

All of these cases are called variable forces, meaning the force changes with displacement. Therefore, we cannot directly use the work equation for constant force.

Graphing for Calculating Variable Work

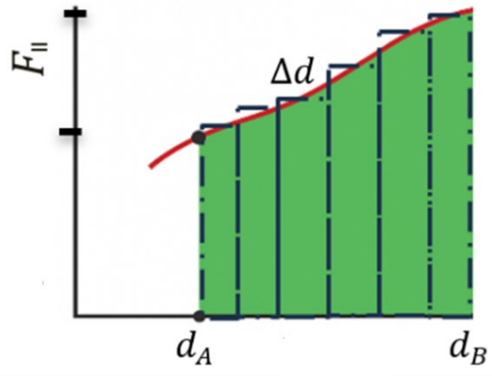
We assume that the force acting on an object is not constant, but rather varies with the displacement d. The component of the force parallel to the displacement can be represented by a graph showing the relationship between $Fcos\theta$ and d, as shown in the figure.



To approximate the work calculation, we divide the total distance into very small segments, each with a magnitude of Δd. We assume that the magnitude of the force in each segment is equal to an average value F_i and the approximate work for that segment is $\Delta w = F_i \, \Delta d$

This is represented geometrically by the area of a small rectangle with a base of Δd and a height of F_i. By adding the partial works for all the sections, we get the total work, that is:

$$W = \sum F_i \, \Delta d \qquad (5)$$

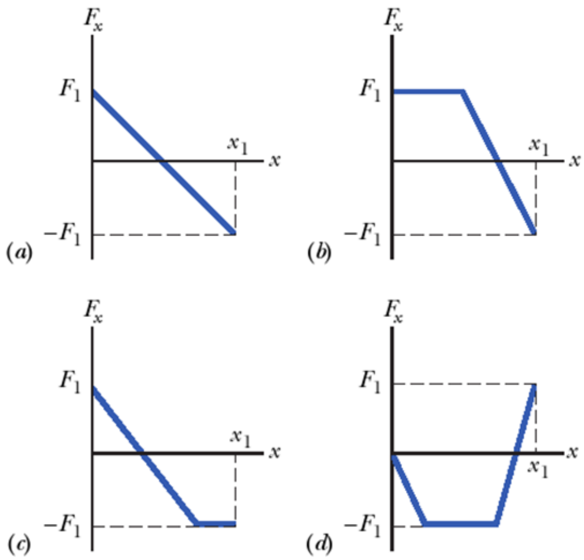


Result

The work calculation graphically equals the area enclosed between the curve (the force component in the direction of motion - position) and the position axis.

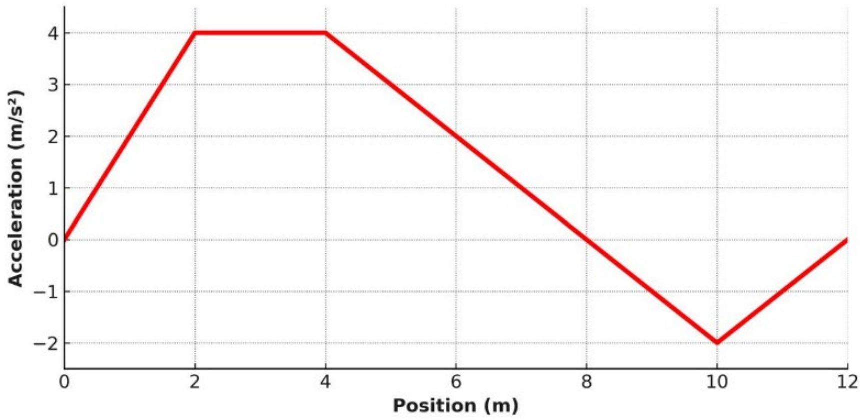
Concept Check

Four graphs (drawn to the same scale) of the x component Fx of a variable force (directed along an x axis) versus the position x of a particle on which the force acts. Rank the graphs according to the work done by the force on the particle from $x = 0$ to $x = x1$, from most positive work first to most negative work last



Exercise 1.3

A mass $2.25kg$ move with the acceleration shown by the graph. find the work done on this mass by the net force?

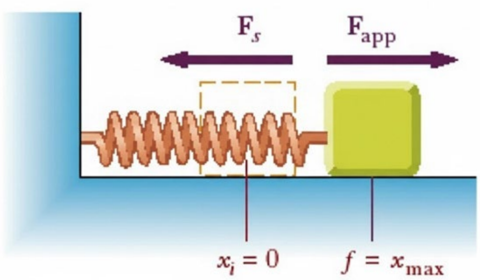


1.1.5 Work Done by a Spring:

Block-Spring System is a common physical system for which the force varies with position is shown in Figure block on a horizontal, frictionless surface is connected to a spring. If the spring is either stretched or compressed a small distance from it unstretched (equilibrium) configuration by applied force F_{app}, the spring exerts on the block a force that can be expressed as:

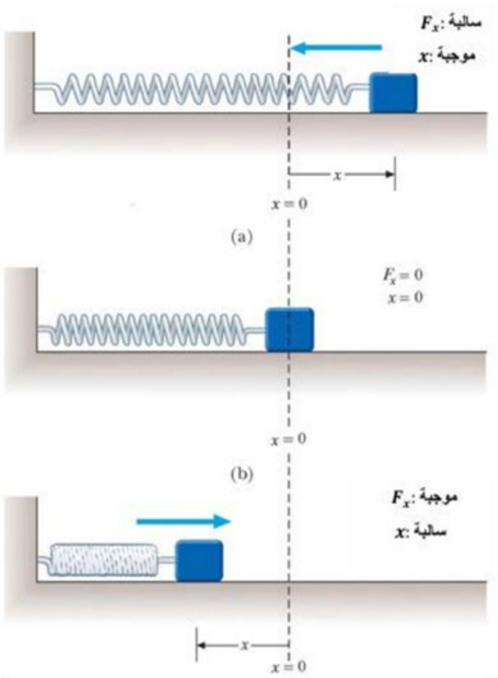
$$F_S = -kx \qquad (6)$$

where x is the position of the block relative to its equilibrium ($x = 0$) position and k is a positive constant called the force constant or the spring constant of the spring, notice that the applied force F_{app} is equal in magnitude and opposite in direction to the spring force $F_S$at all times.

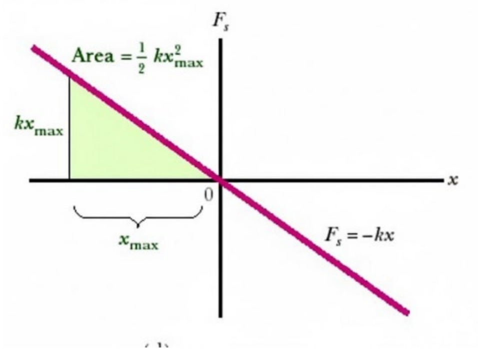


In other words, the force required to stretch or compress a spring is proportional to the amount of stretch or compression X. This force law for springs is known as Hooke's law. The value of k is a measure of the stiffness of the spring. Stiff springs have large k values, and soft springs have small k values. As can be seen from Equation, the units of k are N/m.

The negative sign in Equation signifies that the force exerted by the spring $F_S$is always directed opposite to the displacement from equilibrium$x$.



$$W_{0\to x} = \frac{kx^2}{2}$$



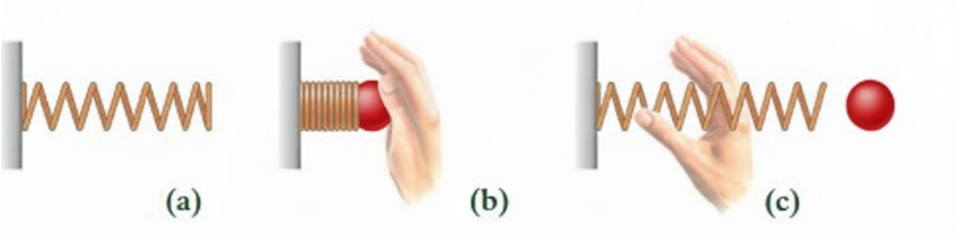
Work is equal to the shaded area between the curve and the x-axis

1.2 ENERGY

The concept of energy is one of the most important topics in science and engineering. In everyday life, we think of energy in terms of fuel for transportation and heating, electricity for lights and appliances, and foods for consumption. However, these ideas do not really define energy. They merely tell us that fuel is needed to do a job and that those fuels provide us with something we call energy.

Energy: One of the Ingredients of the natural world, it takes many forms and can be transformed from one form to another. It can be defined (it is valid for mechanical energy) as the ability to do work, or it is something that causes a change in its surroundings, and it is measured in joules

For example: A spring (a) can store energy (elastic PE) when compressed as in (b) and can-do work when released (c).



1.2.1 Kinetic Energy

Energy acquired by an object due to its motion

$$KE = \frac{1}{2}mv^2 \qquad (7)$$

v $\left(\frac{m}{s}\right)$:velocity

Concept Check

Rank the following velocities according to the kinetic energy a particle will have with each velocity, greatest first:

(a) $\vec{v} = -4\hat{\imath} + 4\hat{\jmath}$

(b) $\vec{v} = -3\hat{\imath} + 4\hat{\jmath}$

(c) $\vec{v} = \frac{5m}{s}.\ \theta = 30°$

Example1.3

Headd-On Collision of Two Locomotives: Two locomotives are placed at opposite ends of a straight 6.40-km-long track. Each locomotive starts from rest and moves toward the other with a constant acceleration of

$$a = 0.26\,\text{m/s}^2.$$

the weight of each locomotive is

$$W = 1.2 \times 10^6\,\text{N}.$$

Assuming both locomotives accelerate uniformly form rest over half of the track length, determine the total kinetic energy of the two locomotives just before collision

the mass of one locomotive is obtained from:

$$m = \frac{W}{g} = \frac{1.2 \times 10^6}{9.80} = 1.22 \times 10^5\,\text{kg}$$

The final velocity is found using

$$v = \sqrt{2ad} = \sqrt{2(0.26)(3.20 \times 10^3)} = 40.8\,\text{m/s}$$

The kinetic energy of one locomotive

$$K = \frac{1}{2}mv^2 = 0.5(1.22 \times 10^5)(40.8)^2 = 1.02 \times 10^8\,\text{J}$$

The total kinetic energy of both locomotives

$$K_{\text{total}} = 2K = 2(1.02 \times 10^8) = 2.04 \times 10^8\,\text{J}$$

1.2.2 Work- Kinetic Energy Theorem:

The laws of conservation of energy and linear momentum have attained broad significance due to the simplicity of their application when analyzing multi-body systems—particularly in situations where determining the force responsible for the motion is difficult or even impossible. It is also noteworthy that these two laws are applicable across various other branches of physics, including atomic physics in its different subfields, especially in cases where Newton's laws of motion lose their validity under such conditions.

According to Newton's second law, when a net force F_{net} (where F_{net} $constant\ force$)acts on a body of mass m, it produces an acceleration

$$a = \frac{v_f^2 - v_i^2}{2d} \qquad (8)$$

By substitution, the work done on the body can be determined as follows:

$$W_{net} = F_{net}d = m(\frac{v_f^2 - v_i^2}{2d}) \times d$$

$$W_{net} = F_{net}d = \frac{1}{2}m(v_f^2 - v_i^2) \quad (9)$$

$$W_{net} = KE_f - KE_i \quad (10)$$

The work–energy theorem states that:

The total work done by the net force acting on a particle is equal to the change in the particle's kinetic energy:

$$\sum \boldsymbol{W} = \boldsymbol{KE_f} - \boldsymbol{KE_i} = \frac{\mathbf{1}}{\mathbf{2}}\,\boldsymbol{m}(\boldsymbol{v_f^2} - \boldsymbol{v_i^2}) \quad (11)$$

Important Notes

-The Work- Kinetic Energy theory can be applied to both constant and variable forces.

-When we use the work-energy theory we must consider all forces that do work on a particle when calculating the total work done.

-The work–kinetic energy theorem indicates that the speed of a particle will increase if the net work done on it is positive, because the final kinetic energy will be greater than the initial kinetic energy.

-The speed will decrease if the net Work is negative, because the final kinetic energy will be less than the initial kinetic energy

Example1.4

A car of mass $m = 1.0 \times 10^3$ kg accelerates uniformly from an initial speed of $v_i = 15.0$ m/s to a final speed of $v_f = 25.0$ m/s. Determine the net work W_{net} required to cause this change in motion.

Solution:

The net work done on an object equals the change in its kinetic energy:

$$W_{net} = \Delta K = \frac{1}{2}m(v_f^2 - v_i^2)$$

Substitution:

$$W_{net} = \frac{1}{2}(1.0 \times 10^3)[(25.0)^2 - (15.0)^2]$$
$$W_{net} = 2.0 \times 10^5 \ J$$

Exercise1.4

A body is released from rest and falls freely through a vertical distance d under the action of gravity alone. Show, using appropriate equations, that the total work done on the body by the gravitational force is equal to the kinetic energy gained by the body during its fall.

Concept Check

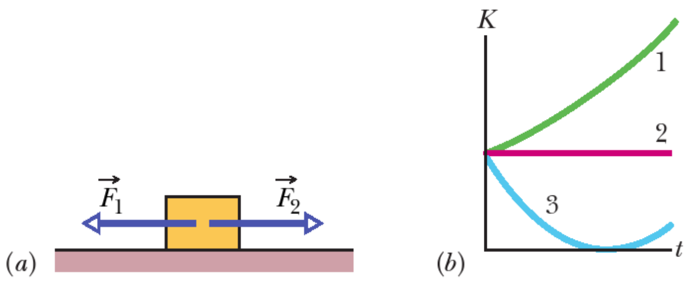
A block slides to the right on a smooth, frictionless horizontal surface while being acted upon by two horizontal forces, F_1 and F_2. The figure displays three plots of the block's kinetic energy K as a function of time t.

Based on the figure, determine which plot corresponds to each of the following cases:

(a) When the two forces are equal in magnitude $F_1 = F_2$.

(b) When the first force is greater than the second $F_1 > F_2$.

(c) When the first force is less than the second $F_1 < F_2$.



1.2.3 Potential Energy

It's the energy associated with the configuration of a system of objects that exert forces on each other, and associated with conservative forces, such as:

Gravitational Potential Energy: It accompanies a system consisting of the Earth and an object and is related to the force of Earth's gravity (a conservative force).

Elastic potential energy: It accompanies a system consisting of a spring and a block attached to it and is related to the spring force (conservative force).

Potential energy can be considered as stored energy that can do work or be converted into kinetic energy.

1.2.4 Gravitational Potential Energy

Energy acquired by an object if it is higher or lower than a reference frame with a vertical displacement h

$$PE_g = mgy \qquad (12)$$

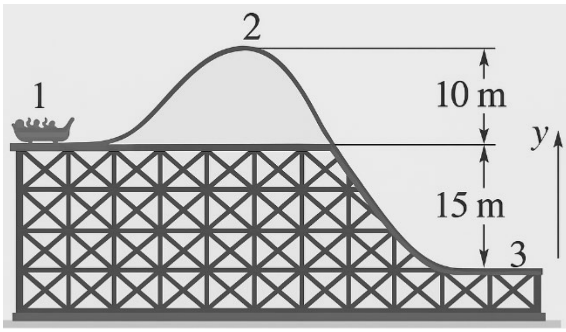
That Equation is valid only for objects near the surface of the Earth, where g is approximately constant.

Example 1.5

A roller-coaster car of mass $m = 1.00 \times 10^3$ kg moves successively from point (1) to point (2) and then to point (3).

(a) Determine the gravitational potential energy of the car at points (2) and (3), taking the reference level $y = 0$at point (1).

(b) Find the change in potential energy as the car moves from point (2) to point (3).



(a) Calculating the Gravitational Potential Energy at Points (2) and (3):

The mass of the roller-coaster car is $m = 1.00 \times 10^3$ kg, and the acceleration due to gravity Is $g = 9.8 m/s^2$.The reference level is taken at point (1), where $y_1 = 0$.

From the figure, the height of point (2) is $y_2 = 10.0$ m, while point (3) is below the reference level at $y_3 = -15.0$ m.

Thus:

$$PE_2 = mgy_2 = (1.00 \times 10^3)(9.80)(10.0) = 9.80 \times 10^4 \text{ J}$$
$$PE_3 = mgy_3 = (1.00 \times 10^3)(9.80)(-15.0) = -1.47 \times 10^5 \text{ J}$$

(b) Calculating the Change in Potential Energy:

$$\Delta PE = PE_3 - PE_2 = (-1.47 \times 10^5) - (9.80 \times 10^4) = -2.45 \times 10^5 \text{ J}$$

The negative sign indicates that the potential energy decreased by 2.45×10^5 J.

This means the car lost gravitational potential energy, which was converted into kinetic energy as it moved downward from point (2) to point (3).

Exercise 1.5

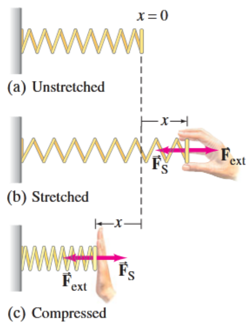
In the previous example (1.2.4), re-solve both parts by taking the reference point $(y = 0)$at point 3.

Does the result change? What do you conclude?

1.2.5 Elastic Potential Energy

Energy stored in a spring due to its compression or expansion from its equilibrium position with value x

$$PE_s = \frac{1}{2} kx^2 \qquad (13)$$



Example 1.6

A spring with a force constant kis stretched by a force F, causing an extension x. Then, a new force 2Fis applied.

What is the ratio of the new elastic potential energy to the previous one? Explain the result you obtain.

Solution:

From the force relation:

When a force F is applied, the spring extends by x:

$$F_1 = kx$$

When a force 2F is applied, the extension becomes 2x:

$$F_2 = k(2x)$$

From the energy relation:

$$PE_1 = \frac{1}{2}kx^2, PE_2 = \frac{1}{2}k(2x)^2$$

Energy ratio:

$$\frac{PE_2}{PE_1} = 4$$

The force is directly proportional to the extension ($F \propto x$),

but the elastic potential energy is proportional to the square of the extension ($PE \propto x^2$).

Therefore, when the force doubles, the extension doubles, and the potential energy increases fourfold.

Conservative And not Conservative Forces:

Properties of Conservative Forces: Conservative forces have two important properties:

(a) The work done by a conservative force on an object moving between any two points does not depend on the motion's path.

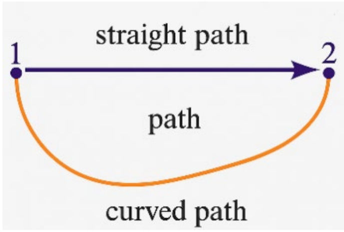
(b) The work done by a conservative force on an object in a closed path of motion is equal to zero

(a closed path is the path where the initial position applies to the final position).

For example: the gravitational force and the spring force are conservative forces. This is clear from the equations for calculating the work of the gravitational force and the force of the spring,

crate is pushed slowly at constant speed across a rough floor from position 1 to position 2 via two paths, one straight and one curved. traveled d is greater (as for the curved path), then work done by friction force is greater. The work done does not depend only on points 1 and 2; it also depends on the path taken.

Therefore: the frictional force is nonconservative force



1.2.6 Conservation Of Mechanical Energy

It can be easier to apply the Work- Kinetic Energy Theory by classifying the mechanical system into conservative or non-conservative.

NON-CONSERVATIVE SYSTEM	CONSERVATIVE SYSTEM
It is affected by non-conservative forces. Note: Forces (friction-external pull and push forces- air resistance) are examples of non-conservative forces. $\sum W_{app} - f_k d = E_f - E_i$ (16) The sum of the external work $\sum W_{app}$ (except the work of conservative forces such as weight and spring force). The work of the kinetic friction force $-f_k d$	It is not affected by non-conservative forces. Note: The (weight-spring) forces are conservative forces. $E_f = E_i$ (14) E_f The sum of the initial and final mechanical energies Mechanical energies mean: Kinetic, Gravitational Potential, Elastic Potential $\frac{1}{2}mv_f^2 + \frac{1}{2}kx_f^2 + mgy_f = \frac{1}{2}mv_i^2 + \frac{1}{2}kx_i^2 + mgy_i$ (15)

the principle of conservation of mechanical energy:

If only conservative forces do work, the total mechanical energy of a system neither increases nor decreases in any process. It stays constant—it is conserved.

Problem –Solving Hints:

1. Draw a picture of the physical situation.

2. Determine the system for which you will apply energy conservation: the object or objects and the forces acting.

3. Ask yourself what quantity you are looking for and choose initial (point 1) and final (point 2) positions.

4. If the object under investigation changes its height during the problem, then choose a reference frame with a convenient Y=0 level for gravitational potential energy; the lowest point in the situation is often a good choice. If springs are involved, choose the unstretched spring position to be x = 0.

5.Apply conservation of energy. For other nonconservative forces use your intuition for the sign of its work is the total mechanical energy increased or decreased in the process.

6. Use the equation(s) you develop to solve for the unknown quantity.

Example 1.7

If a rock is released from an initial height of $h = 3.0$ m, determine its speed when it has fallen to a height of 1.0 mabove the ground, applying the principle of conservation of mechanical energy

Solution:

We apply the principle of conservation of mechanical energy, where the sum of potential and kinetic energies remains constant (neglecting air resistance):

$$PE_1 + KE_1 = PE_2 + KE_2$$

At the moment of release from rest, the initial velocity is $v_1 = 0$, therefore:

$$mgh_1 = mgh_2 + \frac{1}{2}mv_2^2$$

By cancelling the mass from both sides:

$$gh_1 = gh_2 + \frac{1}{2}v_2^2$$

$$v_2 = \sqrt{2g(h_1 - h_2)}$$

Substituting the values:

$$v_2 = \sqrt{2(9.8)(2.0)} = \sqrt{39.2} = 6.26 \text{ m/s}$$

Considering significant figures:

$$v_2 = 6.3 \text{ m/s}$$

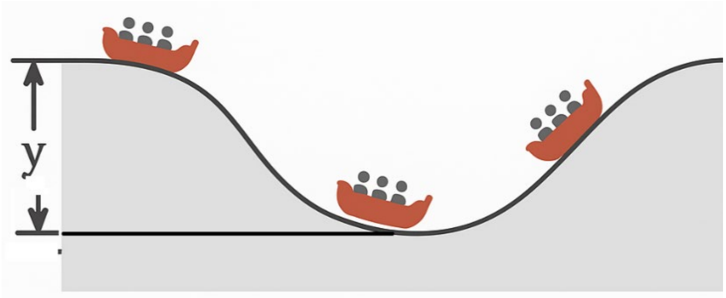
Exercise 1.6

If the height of the hill in the figure is $y = 40$ mand the roller-coaster car starts from rest at the top, determine:

(a) the car's speed at the bottom of the hill, and

(b) the height at which the car's speed is half of that value.

Assume $y = 0$at the bottom of the hill and neglect friction.



1.3 POWER

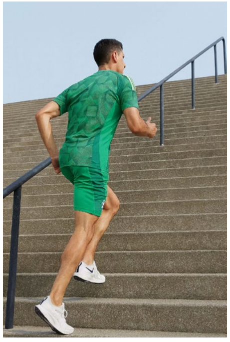
Power is defined as the rate at which work is done. The average power equals the work done divided by the time required to do it .Power can also be defined as the rate of energy transfer. It is a scalar quantity measured in watts (joules per second) in the International System of Units (SI). In practical applications, mechanical horsepower is sometimes used to measure the power of engines and machines, where one horsepower is approximately equal to 746 watts.

1 hp = 746 J/s

INSTANTANEOUS POWER	AVERAGE POWER
Rate of work done (or output) on an object or a machine per time unit If the force is constant in magnitude and direction, its instantaneous power can be calculated by the law: $P = Fv$ (18) If the velocity is constant, then: instantaneous power = average power.	Average work done (or output) on an object or a machine per time unit: $\bar{P} = \frac{W}{\Delta t}$ (17)

Example 1.8

A runner of mass 60 kgtakes 4.0 s to climb a long staircase whose vertical height is 4.5 m



The power is given by

$$P = \frac{mgh}{t}$$

Substitute the values

$$P = \frac{(60)(9.8)(4.5)}{4.0} = 661.5 \text{ W}$$
$$P = 660 \text{ W} = 6.6 \times 10^2 \text{W}$$

Convert to horsepower

$$P = \frac{660}{746} = 0.88 \text{ hp}$$
$$P = 6.6 \times 10^2 \text{ W} = 0.88 \text{ hp}$$

$$E = Pt = 2.6 \times 10^3 J$$

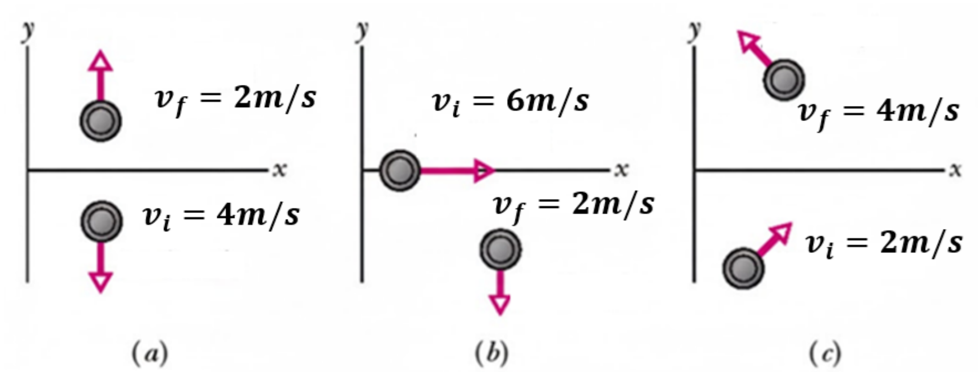
Exercise 1.7

A body of mass 10 kgstarts from rest and moves up a smooth inclined plane (friction neglected) under the action of a force of 96 Nparallel to the incline and directed upward. If the length of the incline is 25 mand its angle of inclination is 37°, calculate the instantaneous power of the force Fat the end of the incline in watts (W).

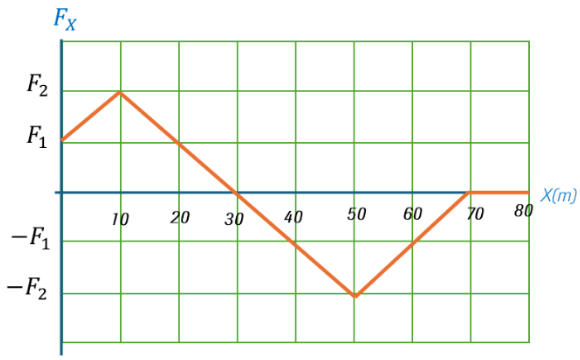
Chapter (1) Questions: Work and Energy

Concept Check:

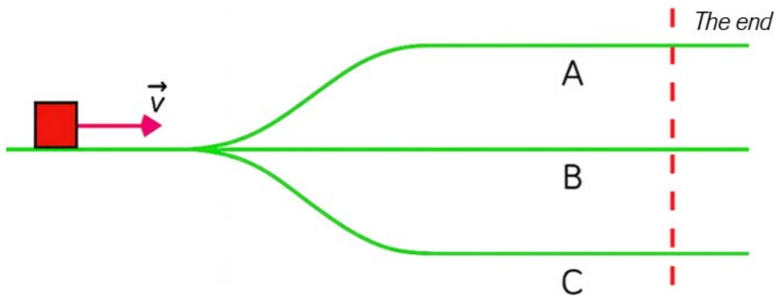
(1) In three situations, a briefly applied horizontal force changes the velocity of a hockey puck that slides over frictionless ice. The overhead views of Fig. ind0icate, for each situation, the puck's initial speed vi, its final speed vf, and the directions of the corresponding velocity vectors. Rank the situations according to the work done on the puck by the applied force, most positive first and most negative last



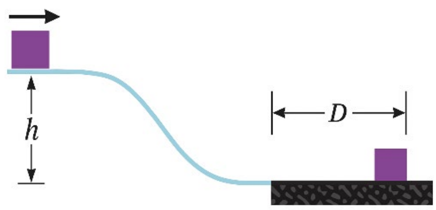
(2) Figure gives the x component F_X of a force that can act on a particle. If the particle begins at rest at $x = 0$, what is its coordinate when it has (a) its greatest kinetic energy, (b) its greatest speed, and (c) zero speed? (d) What is the particle's direction of travel after it reaches $x = 60\ m$?



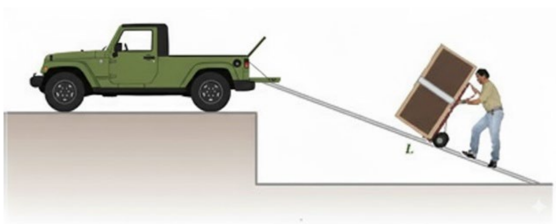
(3) In Figure, a horizontally moving block can take three frictionless routes, differing only in elevation, to reach the dashed finish line. Rank the routes according to (a) the speed of the block at the finish line and (b) the travel time of the block to the finish line, greatest first



(4) In Fig. a block slides along a track that descends through distance h. The track is frictionless except for the lower section. There the block slides to a stop in a certain distance D because of friction. (a) If we decrease h, will the block now slide to a stop in a distance that is greater than, less than, or equal to D? (b) If, instead, we increase the mass of the block, will the stopping distance now be greater than, less than, or equal to D?



(5) A man wishes to load a refrigerator onto a truck using a ramp, as shown in Figure. He claims that less work would be required to load the truck if the length L of the ramp were increased. Is his statement valid?

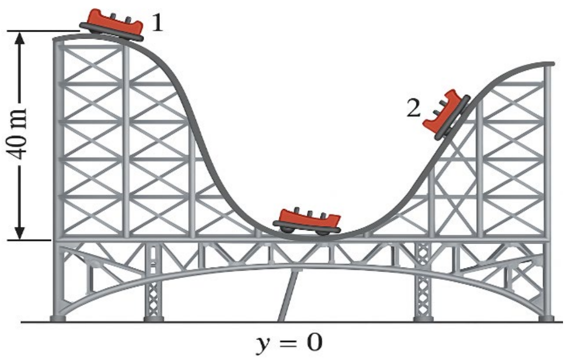


Problems and applications:

(1) A vehicle traveling at 60.0 km/h requires 20 .0m to come to a complete stop. What distance will the vehicle need to stop if it is moving at twice its initial speed, that is, at 120.0 km/h? Assume that the maximum braking force does not depend on the magnitude of the speed.

(2) A 6.0-kg block initially at rest is pulled to the right along a horizontal surface by a constant horizontal force of 12.0N. Find the speed of the block after it has moved 3.0 m if the surfaces in contact have a coefficient of kinetic friction of 0.15

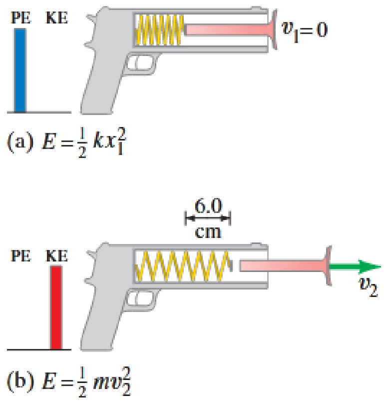
(3) The roller-coaster car reaches a vertical height of only 25.0 m on the second hill, where it slows to a momentary stop. It traveled a total distance of $4.0 \times 10^2\ m$. Determine the average friction force (assume it is roughly constant) on the car, whose mass is $1.0x10^3\ kg$



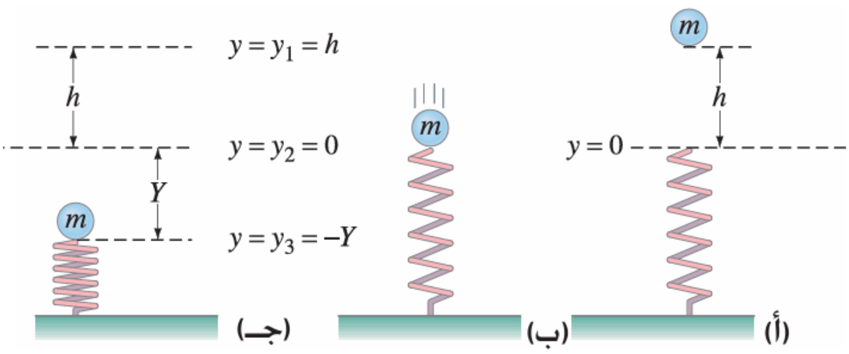
(4) A dart of mass 0.100 kg is pressed against the spring of a toy dart gun as shown in figure. Spring with spring stiffness constant $2.5 \times 10^2 N/m$ and ignorable mass, is compressed 6.0 cm and released. If the dart detaches from the spring when the spring reaches its natural length what speed does the dart acquire? If the dart is thrown vertically: find:

-The maximum height reached by the dart's center of mass.

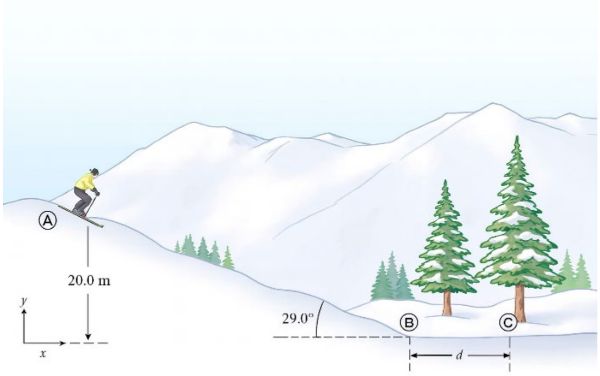
-The velocity of the center of mass of the dart at the moment the spring reaches its natural length.



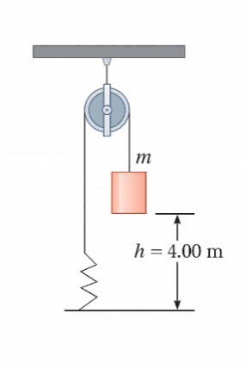
(5) A ball of mass m=2.60 kg starting from rest, all's a vertical distance h=55.0 cm before striking a vertical coiled spring, which it compresses an amount Y= 15.0 cm. Determine the spring stiffness constant k of the spring. Assume spring has negligible mass, and ignore air resistance



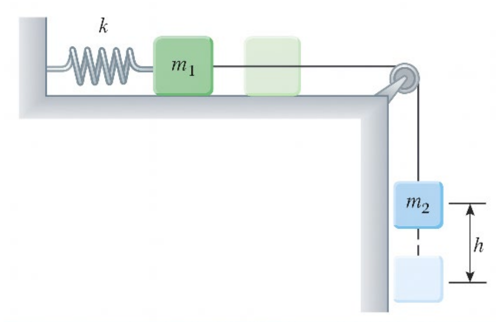
(6) A skier starts from rest at the top of a frictionless incline of height 20.0 m, as shown in Figure. At the bottom of the incline, she encounters a horizontal surface where the coefficient of kinetic friction between the skis and the snow is 0.210. How far does she travel on the horizontal surface before coming to rest, if she simply coasts to a stop? Use the principle of energy conservation.



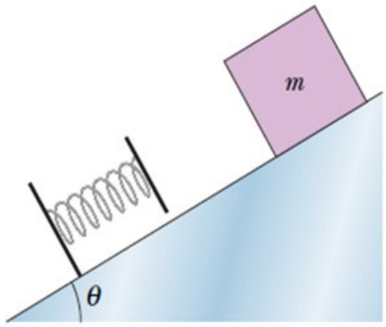
(7) A mass tied with a spring, it's in normal length, the mass falls a distance h before coming to rest, If the spring constant: $4.0 \times 10^2 N/m$ Calculate the value of the mass.



(8) Two blocks are connected by a light string that passes over a frictionless pulley, as shown in Figure. The block of mass m_1 lies on a horizontal surface and is connected to a spring of constant k. The system is released from rest when the spring is unstretched. If the hanging block of mass m_2 falls a distance h before coming to rest, calculate the coefficient of kinetic friction between the block of mass m_1 and the surface



(9) a block of mass m=12 kg is released from rest on a frictionless incline of angle 30°. Below the block is a spring that can be compressed 2.0 cm by a force of 270 N. The block momentarily stops when it compresses the spring by 5.5 cm. (a) How far does the block move down the incline from its rest position to this stopping point? (b) What is the speed of the block just as it touches the spring?



chaptre2 LINEAR MOMENTUM AND COLLISIONS

2.1 LINEAR MOMENTUM

The law of conservation of energy, which we discussed in the previous chapter, is one of the great conservation laws in physics. Among other conserved quantities are linear momentum, angular momentum, and electric charge.
In this chapter, we shall discuss linear momentum and its law of conservation. In fact, the law of conservation of linear momentum is essentially a reformulation of Newton's laws, offering both a deeper physical insight and an effective tool for problem solving.

We will make use of the laws of conservation of linear momentum and energy to analyze collisions. Indeed, the conservation of linear momentum is particularly useful when dealing with a system of two or more interacting bodies, such as in collisions between ordinary macroscopic objects or between atomic and subatomic particles.

2.1.1 Momentum and its relation to force

The linear momentum (or simply momentum) of an object is defined as the product of its mass and its velocity, expressed as

$$p = mv \qquad (19)$$

Since velocity is a vector quantity, momentum is also a vector; it has both magnitude and direction

The direction of momentum is the same as the direction of velocity, and its magnitude is given by

$$p = mv$$

When a particle moves in an arbitrary direction in space, its momentum possesses three components corresponding to the three spatial dimensions, and can be expressed as:

$$P = mv = m(v_x\hat{\imath} + v_y\hat{\jmath} + v_z\hat{k} \qquad (20)$$

or explicitly in component form:

$P_X = mv_x \quad , \quad P_y = mv_y \;\; P = mv_z$

In everyday experience, a fast-moving car has more momentum than a slow-moving car of the same mass, and a heavy truck has greater momentum than a small car moving at the same speed.

The greater the momentum of an object, the more difficult it is to stop it and the greater the impact it produces when brought to rest by collision.

Because velocity depends on the chosen frame of reference, momentum also depends on it. Therefore, the reference frame must always be specified when describing momentum.

The SI unit of momentum is the product of mass and velocity, kilogram meter per second (kg·m/s), which has no special name.

Changing an object's momentum whether increasing, decreasing, or redirecting it requires a force.

In fact, Newton originally formulated his second law in terms of momentum. In modern notation, it can be written as:

Newton's Second Law:

The rate of change of momentum of an object is equal to the net force acting on it

$$\sum F = \frac{\Delta P}{\Delta t} \qquad (21)$$

.This means that the net force applied to an object equals the time rate of change of its momentum.

When the mass of the object is constant, this relation reduces to the familiar form

$$\sum F = ma \qquad (22)$$

However, in systems where mass changes during motion such as rockets ejecting exhaust gases or objects losing mass while moving the more general form

$$\sum F = \frac{\Delta P}{\Delta t}$$

remains the most accurate and useful expression

Example 1.9

A tennis ball is hit during a serve and leaves the racket with a speed of 55 m/s(198 km/h). If the ball's mass is 0.060 kgand it remains in contact with the racket for 4.0×10^{-3} s, calculate the average force exerted on the ball.

Would this force be enough to lift a person of mass 60 kg?

Solution:

The velocity of the ball changes from

$v_1 = 0$to $v_2 = 55$ m/s

in a time interval of

$\Delta t = 4 \times 10^{-3}$ s.

Average force:

$$F = \frac{\Delta p}{\Delta t} = \frac{m(v_2 - v_1)}{\Delta t} = \frac{(0.060)(55-0)}{4 \times 10^{-3}} = 8.3 \times 10^2 N$$

This force is greater than the force required to lift a 60 kg person,

since that would require approximately

$$F = mg = (60)(9.8) \approx 600 \text{ N}.$$

Exercise 1.8

We can readily derive the familiar $\sum \mathbf{F} = m\mathbf{a}$ form of the second law $\sum \mathbf{F} = \frac{\Delta \mathbf{P}}{\Delta t}$ Try to do

Concept Check

Two objects have the same kinetic energy, explain the situations that they do not have the same linear momentum.

2.1.2 Impulse and Linear Momentum

An impulse of a force (During a certain period) is a vector quantity in the same direction of the force and is calculated by:

$$I = F\,\Delta t \qquad (23)$$

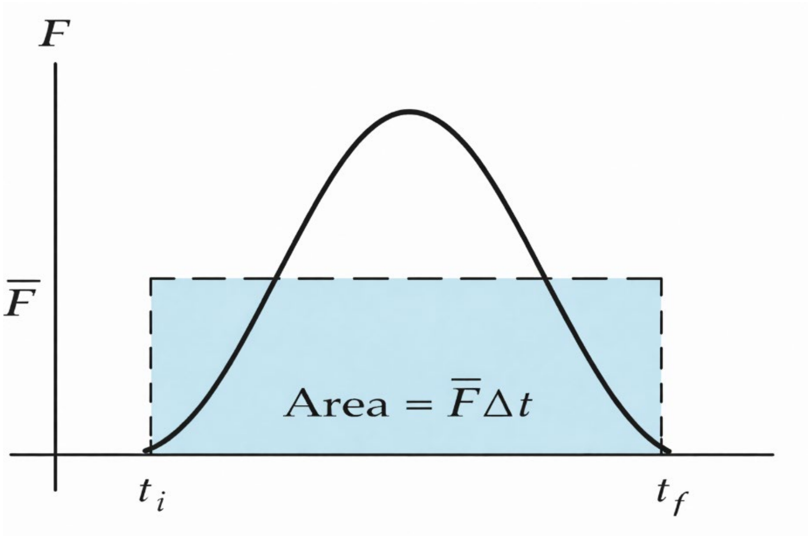
I: Impulse {N.S) F: force (N) Δt :Time of Action of Force(s)

Important Notes

If the force is changing regularly (increasing or decreasing) we take the average force in the impulse calculation: $I = \bar{F}\,\Delta t$

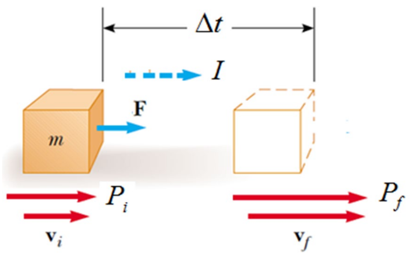
Calculation of impulse graphically:

Equivalent to the area between the curve (force -time) and the time axis



This time-averaged force $\bar{F}$, shown in Figure, can be interpreted as the constant force that would give to the particle in the time interval Δt the same impulse that the time-varying force F gives over this same interval.

2.1.3 Impulse-Momentum Theorem



The impulse of the force acting on a particle equals the change in the momentum of the particle.

$$I = F\,\Delta t = \Delta \boldsymbol{P} = \boldsymbol{m}(\boldsymbol{v}_f - \boldsymbol{v}_i) \qquad (24$$

The direction of the impulse vector is the same as the direction of the change in momentum.

Example1.10

Calculate the impulse produced when a person of mass 70 kg hits a hard ground after jumping from a height of 3.0 m.

Solution:

We consider the person's jump as a free fall with an initial velocity of zero and take the ground level as y = 0. Thus,

$$v^2 = v_1^2 + 2g(\Delta y)$$

$$v = \sqrt{2g(y_0 - y)} = \sqrt{2(9.8\,\text{m/s}^2)(3\,\text{m})} = 7.7\,\text{m/s}$$

The impulse is given by

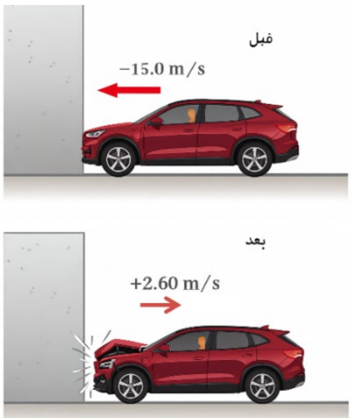
$$F\Delta t = m\Delta v$$

$$F\Delta t = 70kg(7.7m/s - 0) = -5.4 \times 10^2 N.s$$

Exercise 1.9

In a special collision test, a car of mass 1500 kgcrashes into a wall as shown in the figure.

If the car's initial velocity is $\vec{v_i} = (-15\,\hat{\imath})$ m/sand its final velocity after the collision is $\vec{v_f} = (2.6\,\hat{\imath})$ m/s, and the collision lasts 0.150 s, calculate the impulse (or change in momentum) acting on the car.



Concept Check

Airbags in automobiles have saved countless lives in accidents. explain that by using impulse-momentum theorem.

2.1.4 Conservation of Linear Momentum

The total linear momentum of a closed and isolated system (that is, any number of bodies) is conserved. In other words:

Total initial momentum = Total final momentum

$$\sum \overrightarrow{P_i} = \sum \overrightarrow{P_f} \qquad (25$$

This law tells us that the total momentum of an isolated system at every moment is equal to its initial momentum.

It can also be expressed along the three axes:

$$\sum P_{x_i} = \sum P_{x_f} \sum P_{y_i} = \sum P_{y_f} \sum P_{z_i} = \sum P_{z_f}$$

$$P_i = P_f \text{ and } F = 0 = \Delta P = P_f - P_i \text{ and } I = F\Delta t = \Delta P$$

Closed system: One that neither gains nor loses mass.

Isolated system: One on which the net external force is zero.

Notice that we did not specify the nature of the forces acting on the objects in the system. The only requirement of this law is that these forces must be internal to the system.

Example 1.11

Determine the recoil velocity of a rifle with a mass of 5.0 kgwhen it fires a bullet of mass 0.020 kgat a speed of 620 m/s.

Solution:

Momentum after the explosion = Momentum before the explosion

$$m_B v_B + m_R v_R = m_B v'_B + m_R v'_R$$

Initially:

$$0 + 0 = m_B v'_B + m_R v'_R$$

Recoil velocity of the rifle:

$$v_R' = -\frac{m_B v_B'}{m_R} = -2.5 \text{ m/s}$$

Exercise 1.10

An Eskimo hunter with a mass of 60 kgstands at rest on smooth, frictionless ice and fires an arrow of mass 0.50 kghorizontally at a speed of 50 m/s.

Calculate the recoil velocity of the hunter on the ice after firing the arrow.

2.2 COLLISIONS

Momentum is conserved in any collision: The total momentum of an isolated system immediately before a collision is equal to the total momentum of the system immediately after the collision.

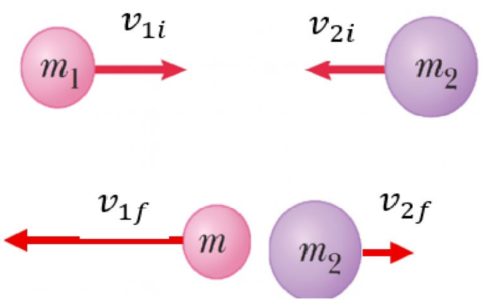
This is expected because no external forces act on the system; the forces involved are internal forces only.

Collisions are classified into two types: Elastic collisions, in which energy is conserved before and after the collision.

Inelastic collisions, in which only momentum is conserved, while energy is not conserved after the collision.

2.2.1 Elastic Collision in One Dimension

Elastic collisions are studied by applying the laws of conservation of linear momentum and conservation of kinetic energy to two small bodies that collide directly, with their motion confined to a single straight line.



Let the two bodies A and B move before collision with initial velocities v_A and v_B, respectively. After collision, their velocities become v_A' and v_B'. The positive direction is taken to the right, so $v > 0$ if the body moves to the right and $v < 0$ if it moves to the left.

Law of Conservation of Momentum

This law states that the total momentum of the two bodies before collision is equal to their total momentum after collision:

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B' \qquad (26)$$

Law of Conservation of Kinetic Energy

In an elastic collision, the total kinetic energy of the system is also conserved:

$$\frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 = \frac{1}{2} m_A v_A'^2 + \frac{1}{2} m_B v_B'^2 \qquad (27)$$

Derivation of the Relation Between Velocities

We have two equations governing the motion (momentum and kinetic energy) and two unknowns, v_A' and v_B'.

By subtracting the momentum equation before and after the collision, we get:

$$m_A(v_A - v_A') = m_B(v_B' - v_B)$$

From the kinetic energy equation:

$$m_A(v_A^2 - v_A'^2) = m_B(v_B'^2 - v_B^2)$$

Using the algebraic identity $a^2 - b^2 = (a - b)(a + b)$, we obtain:

$$m_A(v_A - v'_A)(v_A + v'_A) = m_B(v'_B - v_B)(v'_B + v_B)$$

Dividing the second equation by the first gives:

$$v_A + v'_A = v_B + v'_B$$

or equivalently:

$$v_A - v_B = -(v'_A - v'_B) \qquad (28)$$

Important Result

From the above relation, we conclude that the relative velocity of approach before the collision equals in magnitude the relative velocity of separation after the collision, but in the opposite direction:

$$v_A - v_B = -(v'_A - v'_B)$$

This result characterizes elastic collisions in one dimension and is frequently used to simplify calculations without directly applying the kinetic energy equation.

Example1.12

A billiard ball $A$of mass $m$moves with speed $v$and collides directly with another ball $B$of equal mass initially at rest ($v_B = 0$).

Find the velocities of both balls after the collision, assuming it is perfectly elastic.

Solution:

Since $m_A = m_B = m$ and $v_B = 0$, the momentum equation becomes:

$$mv = mv'_A + mv'_B$$

Dividing both sides by m:

$$v = v'_A + v'_B$$

This is the first equation.

From conservation of kinetic energy:

$$\frac{1}{2}mv^2 = \frac{1}{2}mv'^2_A + \frac{1}{2}mv'^2_B$$

Simplifying (cancelling $\frac{1}{2}m$):

$$v^2 = v'^2_A + v'^2_B$$

Alternatively, using the elastic collision condition:

$$v - v_B = -(v'_A - v'_B)$$

Substituting $v_B = 0$:

$$v = v'_B - v'_A$$

Solving the two equations simultaneously gives:

$$v'_A = 0, v'_B = v$$

Result:

Ball Acomes to rest after the collision, while ball Bmoves with the initial velocity of ball A. Thus, the stationary ball acquires the original speed of the moving ball.

Note:

This behavior is commonly observed in billiard-ball collisions when the two balls have equal masses and no noticeable spin.

If the masses differ or rotational motion is present, the velocity transfer will not be complete.

2.2.2 Inelastic Collisions

For inelastic collisions, only the law of conservation of momentum applies, while the total kinetic energy is not conserved.

This is because a portion of the kinetic energy is converted into other forms of energy such as heat, sound, or deformation of the colliding bodies.

Law of Momentum Conservation: $m_A v_A + m_B v_B = m_A v'_A + m_B v'_B$

Change in Kinetic Energy

The decrease in total kinetic energy is given by:

$$\Delta K = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 - \frac{1}{2}(m_A + m_B) v'^2 \qquad (29)$$

This difference represents the amount of energy lost, transformed into heat, deformation, or sound vibrations.

Example1.13

Perfectly Inelastic Collision A cart of mass $m_1 = 3.0$ kg moves with a velocity $v_1 = 4.0$ m/sand collides with a stationary cart of mass $m_2 = 2.0$ kg. If the two carts stick together after the collision, determine:

(a) their common velocity, and (b) the loss in kinetic energy.

Solution:

(a) From the momentum equation:

$$v' = \frac{(3)(4) + (2)(0)}{3 + 2} = 2.4 \text{ m/s}$$

(b) Kinetic energy before collision:

$$K_i = \frac{1}{2}(3)(4^2) = 24 \text{ J}$$

Kinetic energy after collision:

$$K_f = \frac{1}{2}(5)(2.4^2) = 14.4 \text{ J}$$

Loss in kinetic energy:

$$\Delta K = 9.6 \text{ J}$$

In a perfectly inelastic collision, the two carts move together with a common velocity of 2.4 m/s after the collision, and an amount of 9.6 J of kinetic energy is lost due to its transformation into other forms of energy such as heat and deformation.

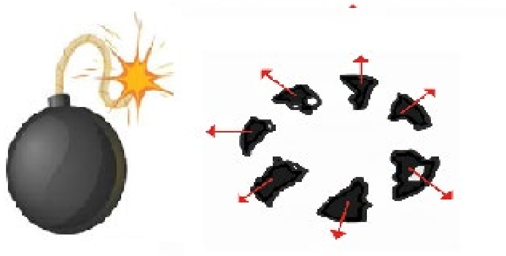
Important Notes

-Elastic and perfectly inelastic collisions are limiting cases; most collisions fall somewhere between them.

-if we take as our system a falling rock, it does not conserve momentum because an external force, the force of gravity exerted by the Earth, accelerates the rock and changes its momentum. However, if we include the Earth in the system, the total momentum of rock plus Earth is conserved. (This means that the Earth comes up to meet the rock. But the Earth's mass is so great, its upward velocity is very tiny.)

-Although there are external forces in the collision systems such as the force of friction and the force of gravity, we can neglect them during the very short collision period, and the law of conservation of momentum becomes applicable before and immediately after the collision.

Fission: like a shell explosion Since the projectile was static before the explosion, so the sum of the momentum of all the shrapnel must be zero.



Concept Check

Let's study some special situations using the above equations, for two objects colliding head-to-head:

The two objects have the same masses (happens in billiard balls):

-If the mass of the first object is much greater than the second, and the second object is at rest (such as the collision of a heavy atom such as uranium with a light atom such as hydrogen):

-If the mass of the second object is much greater than the mass of the first object, and the second object was initially stationary:

Important Notes

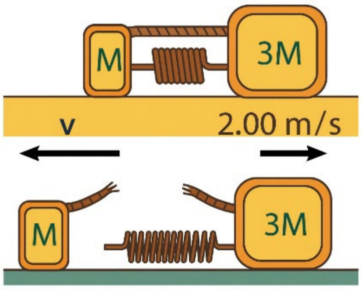
1) The question may specify the type of collision, or you may infer it from a loss of energy during the collision.

2) Pay attention to the speed signals when compensating, in the event of finding an unknown speed, leave its positive signs even if you know that it is in the negative direction, as its correct signal will appear with the final calculations

Exercise 1.11

Two masses rest on a smooth surface, connected by a compressed spring. When the thread holding them together burns, the larger mass is released and moves away with a certain velocity.

If the given data are known, calculate the elastic potential energy stored in the spring. Taking M=1.0kg



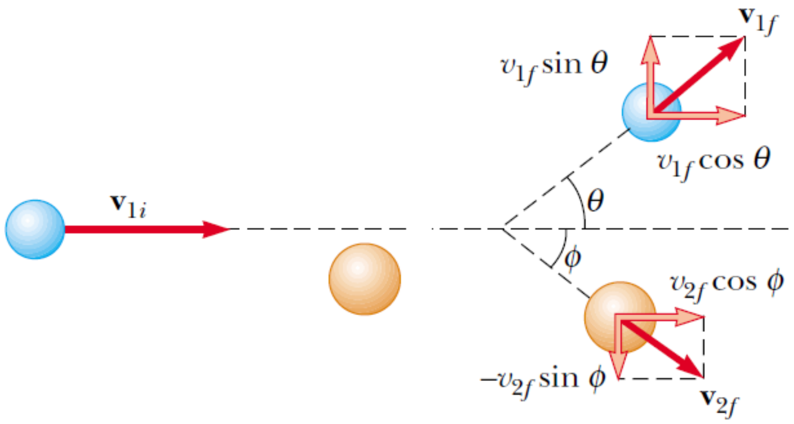
2.2.3 2-D Collision

Let us consider a two-dimensional collision in which particle 1, of mass m_1, collides with particle 2, of mass m_2, which is initially at rest, as shown in the figure.

After the collision, particle 1 moves at an angle θ with respect to the horizontal, while particle 2 moves at an angle $\varphi$with respect to the horizontal.

This type of interaction is known as a glancing collision.

Now, we will apply the laws of conservation of momentum for both the horizontal (x) and vertical (y) components together with the law of conservation of kinetic energy to analyze this type of collision.



Conservation of momentum in the horizontal direction (x-axis):

$$m_1 v_{1i} = m_1 v_{1f} \cos\,\theta + m_2 v_{2f} \cos\,\emptyset$$

Conservation of momentum in the vertical direction (y-axis):

$$0 = m_1 v_{1f} \sin\,\theta - m_2 v_{2f} \sin\emptyset$$

(The negative sign indicates that the two particles move in opposite directions relative to the horizontal axis.)

Conservation of kinetic energy (for an elastic collision):

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

Problem –Solving Hints

Two-Dimensional Collisions: The following procedure is recommended when dealing with problems involving two-dimensional collisions between two objects:

- Set up a coordinate system and define your velocities with respect to that system. It is usually convenient to have the x axis coincide with one of the initial velocities.

- In your sketch of the coordinate system, draw and label all velocity vectors and include all the given information.

- Write expressions for the x and y components of the momentum of each object before and after the collision. Remember to include the appropriate signs for the components of the velocity vectors.

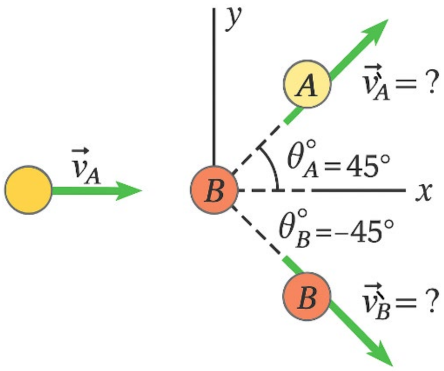
- Write expressions for the total momentum of the system in X direction before and after the collision and equate the two. Repeat this procedure for the total momentum of the system in Y direction.

- If the collision is inelastic, kinetic energy of the system is not conserved, and additional information is probably required. If the collision is perfectly inelastic, the final velocities of the two objects are equal. Solve the momentum equations for the unknown quantities.

- If the collision is elastic, kinetic energy of the system is conserved, and you can equate the total kinetic energy before the collision to the total kinetic energy after the collision to obtain an additional relationship between the velocities.

Example1.14

A billiard ball A is moving with a velocity of $V_A = 3.0$ m/s along the +x-axis and collides with another ball B of equal mass that is initially at rest.
After the collision, both balls move at 45° angles to the x-axis—ball A above the axis and ball B below it as shown in the figure. Determine the speeds of both balls after the collision.



Solution:
We apply the law of conservation of momentum by analysing the motion in two dimensions along the X and Y axes, noting that before the collision, motion exists only along the X-axis.
On the X-axis:

$$mv_A = mv_A' \cos(45°) + mv_B' \cos(-45°)$$

On the Y-axis:

$$0 = mv_A' \sin(45°) + mv_B' \sin(-45°)$$

Since the masses are equal, they cancel out from both equations.
We know that:

$$\sin(-\theta) = -\sin(\theta), \cos(-\theta) = \cos(\theta)$$

Thus:

$$v_B' = v_A'$$

Substituting into the X-axis equation:

$$v_A = 2v_A' \cos(45°)$$

$$v_A' = v_B' = \frac{v_A}{2\cos(45°)} = \frac{3.0 \text{ m/s}}{2(0.707)} = 2.1 \text{ m/s}$$

Therefore, both balls move at a speed of approximately 2.1 m/safter the collision.

Exercise1.12

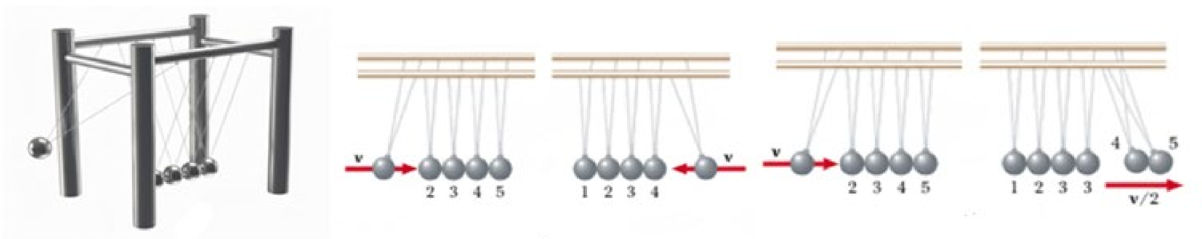
A car of mass $m_1 = 1000$ kg is moving eastward with a velocity of $v_1 = 20$ m/s. It collides with a truck of mass $m_2 = 2500$ kg that is traveling at a velocity of $v_2 = 12$ m/sin a direction 30° north of east. After the collision, the car and truck stick together and move as a single body.

Determine their combined velocity $v_f$and the angle $\theta$the resulting motion makes with the east direction…

Chapter Questions (2): Linear Momentum

Check Your Understanding

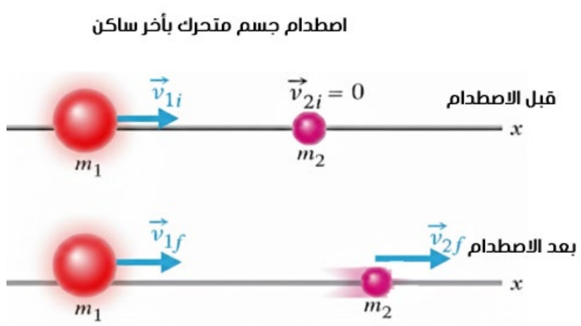
(1) An ingenious device that illustrates conservation of momentum and kinetic energy is shown in Figure. It consists of five identical hard balls supported by strings of equal lengths. When ball 1 is pulled out and released, after the almost-elastic collision between it and ball 2, ball 5 moves out, as shown in Figure b. If balls 1 and 2 are pulled out and released, balls 4 and 5 swing out, and so forth. Is it ever possible that when ball 1 is released, balls 4 and 5 will swing out on the opposite side and travel with half the speed of ball 1, as in Figure c?



a b c

(2) Determine the final linear momentum of the target shown in the figure if the initial linear momentum of the projectile is $6.0 kg.m/s$ and its final linear momentum is

(a) 2.0 kg·m/s
(b) −2 .0 kg·m/s?
(c) What is the final kinetic energy of the target if the initial and final kinetic energies of the projectile are 5.0J and 2.0J, respectively?

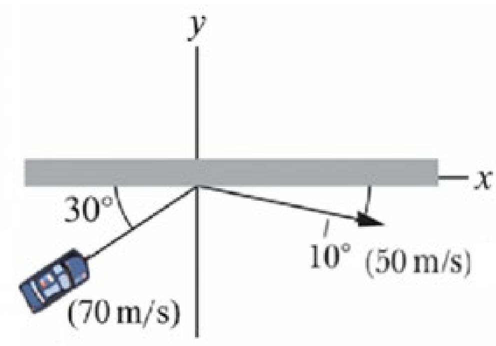


Problems and applications:

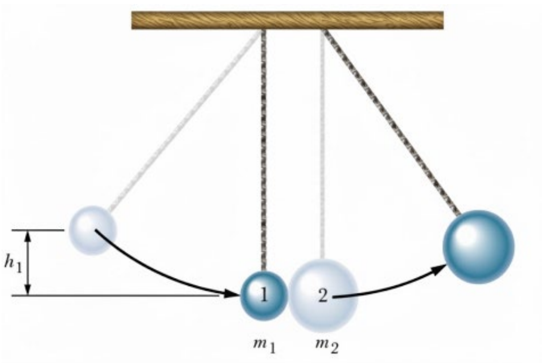
(1) Race Car–Wall Collision: The figure shows a top view of the path taken by a race car driver as his car collides with the racetrack wall. Just before the collision, the driver is moving at a speed of $v_i = 70.0$ m/s along a straight line making an angle of 30.0° with the wall. Immediately after the collision, he is traveling at a speed of $v_f = 50.0$ m/s along a straight line making an angle of 10.0° with the wall. The driver's mass is $m = 80.0$ kg. Questions:

What is the impulse on the driver due to the collision?

If the collision lasts for $14\ ms$, what is the magnitude of the average force on the driver during the collision?

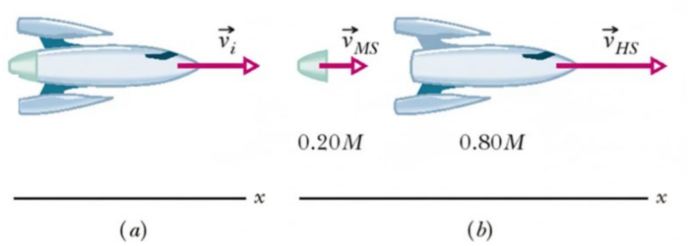


(2) Two metal spheres are suspended from vertical strings and initially just touch, as shown in the figure. Sphere 1, with a mass of m_1 = 30.0 g, is pulled to the left to a height of h_1 = 8.0 cm and then released from rest. After swinging downward, it undergoes an elastic collision with sphere 2, which has a mass of m_2 = 75.0 g. Find the velocity (v_{1f}) of sphere 1 just after the collision.

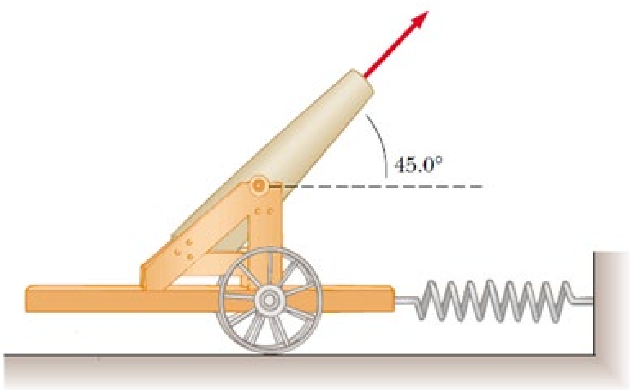


(3) Figure shows a space hauler and cargo module, of total mass M, traveling along an x axis in deep space. They have an initial velocity of magnitude $2.1 \times 10^3\ km/h$ relative to the Sun. With a small explosion, the hauler ejects the cargo module, of mass 0.20M. The hauler

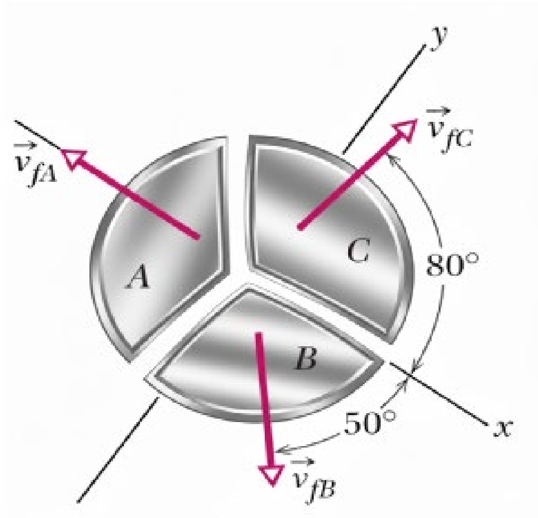
then travels $5.0 \times 10^2\ km/h$ faster than the module along the x axis; that is, the relative speed v_{rel} between the hauler and the module is 500 km/h. What then is the velocity of the hauler relative to the Sun?



(4) A cannon is rigidly attached to a carriage, which can move along horizontal rails but is connected to a post by a large spring, initially unstretched and with force constant $2.0 \times 10^2 kg$, as in Figure. The cannon fires a -200kg projectile at a velocity of 125 m/s directed 45.0° above the horizontal. (a) If the mass of the cannon and its carriage is $5.0 \times 10^3 kg$, find the recoil speed of the cannon. (b) Determine the maximum extension of spring. (c) Find the maximum force the spring exerts on the carriage. (d) Consider the system consisting of the cannon, carriage, and shell. Is the momentum of this system con served during the firing? Why or why not?



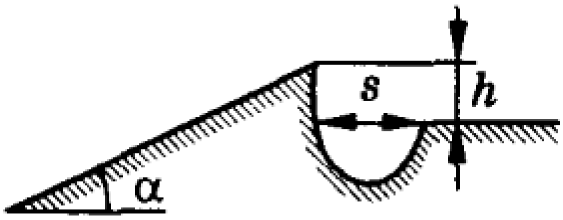
(5) A firecracker placed inside a coconut of mass M, initially at rest on a frictionless floor, blows the coconut into three pieces that slide across the floor. An overhead view is shown in Figure. Piece C, with mass 0.30M, has final speed $v_{Fc} = 5.0\ m/s$. (a) What is the speed of piece B, with mass $0.20M$?(b) What is the speed of piece A?



y
$\vec{v}_{fA}$
$\vec{v}_{fC}$
A
C
80°
B
50°
x
$\vec{v}_{fB}$

SIMULATION TEST FOR STAGE THREE

Question 1 (2p): A motorbike drives onto the high bank of the river (Fig). What is the minimum speed a motorcyclist should have at the moment of breaking away from the left bank in order to jump over the river? The angle α and distances s, h are given.

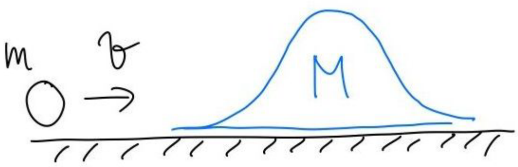


Question 2 (1p): Two identical balls each with mass m are connected by a thread. This system moves with velocity v on a horizontal smooth table. The velocity is perpendicular to the thread. Thread hits the nail in the middle. What is the tension of the thread immediately after that?

Question 3 (1p): The motionless body breaks into 2 fragments with masses m and $2m$. The total kinetic energy of the fragments is E. Find the velocity of a fragment with mass m.

Question 4 (3p): A test tube of length L and mass M lies on a smooth horizontal table. A ball with mass m flies into the tube with a velocity v_0 along the axis. After elastic impact with the inner end the ball pops out of the test tube. How long has the ball been inside the tube? Neglect the rotation of the ball.

Question 5 (2p): A smooth slide of height h and mass M can slide along a smooth horizontal plane. A small washer with mass m slides along the plane with initial velocity v. Find the minimum value of velocity v when the washer will overcome the slide?



Question 6 (4p): Consider a smooth fixed sphere with radius R. A small ball starts moving from the top of the sphere with negligible initial velocity. At some point the ball detaches from the sphere. Find the position of this point.

Solutions

Exercises:

Chaptre1

(1) $W = 5.1 \times 10^2$ J

(2) $|\vec{F}| = 6.3$ N, $|\vec{d}| = 3.0$ m, $\theta = 110°$, $W = -6.0$ J

(3) $W_{\text{net}} = m \times A_{\text{total}} = 2.25 \times 16 = 36$ J

(4) $W = \frac{1}{2} mv^2$

(5) $\Delta PE = PE_3' - PE_2' = 0 - 2.45 \times 10^5 = -2.45 \times 10^5$ J

(6) $v = 28 m/s$, $y = 30$ m

(7) $P = 1.3 \times 10^3$ W

Chaptre2

(1) $\sum F = \frac{\Delta p}{\Delta t}$

(2) $I = \Delta p = 2.64 \times 10^4$ N.s

(3) $v_h = -0.42$ m/s

(4) $PE_s = 24.0 J$

(5) $v_f = 14$m/s , $\theta = 18°$

Solutions to end-of-chapter questions

Chptre1

Concept Check

(1) $W_c > W_a > W_b$

(2)

(a) its greatest kinetic energy $at\ X = 30m$

(b) its greatest speed $at\ X = 30m$

(c) zero speed $at\ X = 60m$

the particle's direction $x = 60\ m$ $+x$

(3) (a) Order of final speeds: $v_C > v_B > v_A$

(b) Order of arrival (who reaches first): $C \to B \to A$

(4) (a) Decreasing the height h The car will stop before point D.

(b) Increasing the mass the car will stop at the same point D.

(5) It does not depend on the length of the path

Problems and applications:

(1) 80.0 m

(2) 1.8 m/s

(3) $f_k = 3.7 \times 10^2$ N

(4) $v = 3.0$ m/s , $h = 0.46$ m , $v = 2.8$ m/s

(5) $k = 1.59 \times 10^3$ N/m

(6) $d = 95.2$ m

(7) $m = 8.2 \times 10^1$ kg

(8) $\mu_k = \frac{m_2 g - \frac{1}{2}kh}{m_1 g}$

(9) $h = 0.35$ m , $v = 1.7$ m/s

(10) $P = 1.3 \times 10^3$ W

Chapter2

Concept Check

(1) It cannot occur $K_{\text{initial}} = \frac{1}{2}mv^2$, $K_{\text{final}} = \frac{1}{4}mv^2$

(2) $P_{2f} = 4.0 kg.m/s^2$, $P_{2f} = 8.0 kg.m/s^2$, $KE_{2f} = 3.0J$

Problems and applications:

(1) $I = 4 \times 10^3\ N \cdot s$ $F_{avg} = 3 \times 10^5 N$

(2) $v_{1f} = -0.54$ m/s

(3) $v_s = 2.0 \times 10^3$ km/h

(4) $v_C = -3.5$ m/s $\quad x_{\max} = 1.8$ m $\quad F_{\max} = 3.5 \times 10^4$ N